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Abstract

This paper employs a lifecycle model from the consumption-savings literature to examine the tradeoffs between defined benefit and defined contribution pension plans. We examine the effects of varying risk aversion, varying initial income and financial wealth, and varying wage processes (that may be correlated with returns on the risky asset).

Results indicate that wage-indexed claims are not an optimal vehicle for retirement policy if the decision to participate is made early in life, because individuals hold most of their wealth in their human capital and would not wish to increase their exposure to income shocks. Later in life, after most of a worker's human capital has been converted to financial assets, defined benefit pension plans help increase diversification by reducing exposure to financial market risk. The access that defined benefit plans provide to annuities markets and possible guaranteed rates of return over the risk-free rate increase the value of defined benefit plans to workers. The model also predicts that wage-indexed claims will be more valuable when equity markets provide low expected returns or are highly variable and when annuity markets are inefficient.

The model illustrates two economic functions performed by defined benefit plans. Firstly, DB plans pool individual wage risks. This allows older workers to buy a wage-linked security that increases their exposure to wage risks. Secondly, they create a group annuities market that reduces the cost of adverse selection.

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1. Introduction

Most financially active people (and their dependants) face risk from two sources: wage instability and asset return variability. The relative importance of these two sources of risk, and individuals' responses to them, is an issue that has occupied a great deal of the economics literature. The fact that this issue is significant in the context of retirement planning is reflected in the design of retirement plans themselves, of which two major types exist: defined benefit (DB) plans, and defined contribution (DC) plans.

A DB plan pays an individual a benefit defined by some formula, usually without reference either to the amount of contributions the worker has made to the plan or to the level of investment returns that the pension plan has earned on its assets. A typical DB plan pays benefits linked to a worker's final salary and the length of his service with the employer sponsoring the plan. This linkage makes wage variability an important aspect to consider when examining the desirability of DB plans.

DC plans, one the other hand, usually have fixed contributions but variable benefits. The benefits depend on the level of contributions made by the employee and the investment return earned by the assets over the employee's lifetime. Investment risk is thus crucial in trying to understand the role played by DC plans in retirement portfolios.

Both DB and DC plans are common in the United States and elsewhere. Prominent examples of DB plans include the US Social Security system and final salary occupational pension plans, which are often provided to employees of larger US corporations and federal, state and local governments.

A common type of DC plan in the US is the 401(k) plan, where contributions are paid into an individual member's account, often by both the employer and the employee, returns accrue over the employee's working life and the employee can access the accumulated balance from retirement. Both DB and DC plans usually have vesting provisions whereby some benefits (usually only those relating to the employer portion of the contributions) are only available to the worker after a certain number of years of service have been completed.

The balance of pension provision between DC and DB plans has changed dramatically internationally in the last two decades. As recently as 1985, Ippolito (1985) was able to report that 'most pension-covered workers in the United States are covered solely or primarily by defined benefit plans'. By contrast, the US Department of Labor (2001) reported that of the 59% of employees covered by pension plans in 1996-1998, only 59% were members of defined benefit, or DB, plans. Many of these individuals were also members of DC plans. Large differences in pension plan provision have also emerged between public and private sector workers – only 32% of full-time private sector workers were covered by DB plans, while 90% of full-time public-sector workers were members of DB plans.

Although the balance between DB and DC pension coverage has shifted, the number of workers that are members of both DB and DC plans, as well as the stability of DB pension coverage in some sectors of the economy, suggest that both types of plan play an important role in providing retirement security to workers.

Until the late 1980's, the traditional view was that DB plans protected workers from investment risk, as investment risk was borne by plan sponsors, and from income risk, as DB pension payments are explicitly linked to the final level of wages in most DB plans.[1] However, an analysis by Bodie *et al.* (1988) revealed that the dependence of the retirement benefit stream on final wages actually *increased* an individual's exposure to wage risks. This was shown by their conclusion that if wage risks dominated investment risks, risk-averse individuals would prefer DC pension plans over DB plans under certain conditions. However, they included income risk in retirement by recognizing that members of DC plans only have the option to annuitize their wealth at random interest

[1] Some DB plans pay benefits fixed in nominal terms for each year of service. These plans (which are almost all union plans) have an implicit link to wages because the benefits are increased in each round of union negotiations. See Ippolito (1985) for a discussion of union plans.

rates and were unable to reach a conclusion about which plan type a risk-averse individual would prefer.

This paper extends the analysis of Bodie *et al.* (1988) in order to explore the conditions under which risk-averse individuals might prefer one type of pension plan over another. In an environment where many individuals have a mix of pension types, it is also useful to understand the determinants of the optimal mix between DB and DC pensions. To achieve these goals, this paper applies a lifetime savings-consumption model to the DB/DC pension problem. Wages are assumed to be risky and unhedgeable, and individuals are given the option of joining a DB pension plan (and to choose its generosity). Individuals can choose their consumption and asset allocation (between risky and risk-free assets) at each point in their life-cycles. Unlike the earlier Bodie analysis, this model makes wage and investment processes explicit (and possibly correlated), allows wage uncertainty and investment uncertainty to accumulate over time, models mortality both before and after retirement, and allows investment choice to change dynamically and endogenously. This model flexibility is achieved at the expense of analytical tractability, and forces reliance on numerical solutions. The approach of Carroll (1992,1997a,1997b) for numerically solving stochastic dynamic programming problems is used. Interest rates are assumed to be fixed.

A final difference between this paper and earlier research is how we model the DB plan. Bodie *et al.* (1988) based their model of the DB plan on their view of the corporate liability assumed by the plan sponsor. This view has some implications for the labor market that have not been demonstrated empirically. By recognizing the existence of implicit contracts in the labor market, we are able to use a contribution schedule for the DB plan that we believe is more realistic than the one assumed by Bodie *et al.* (1988).

The lifecycle model that our paper uses draws on the consumption-savings literature launched by Deaton (1991). He examined infinite-horizon models of consumption where wage risk is unhedgeable and possibly autocorrelated. Many different authors have used similar models to examine issues in consumption and saving in the presence of unhedgeable background risk. Carroll (1992,1997a,1997b) and Carroll and Samwick (1998) examine 'buffer-stock' savings behaviour and the interest elasticity of saving; Heaton and Lucas (1996, 1997) look at the impact of unhedgeable background risk on portfolio choice; Hubbard, Skinner and Zeldes (1995) investigate the impact of a social welfare system on individual saving; and Campbell *et al.* (2000) study risk aspects of an investment-based social security system. In all these studies, the authors model individuals making decisions about consumption and saving given the fact that they face substantial labor income risk that is unhedgeable due to incomplete financial markets. The models of Carroll and Hubbard, Skinner and Zeldes do not allow for a risky asset or an endogenous asset mix. The focus of the present paper is on the flow of risks through available institutional mechanisms of retirement saving, making it essential to use a model that was designed to examine the issue of retirement saving and that allows for investment risk. This makes the model of Campbell *et al.* (2000) a natural choice. The models of Heaton and Lucas (1996, 1997, 2000) use a similar approach (although their focus is more on consumption and saving than specifically on retirement) and should therefore give answers that are qualitatively similar.

The present paper is a first step on the road towards the ultimate goal of designing and fitting a model to explain the wide diversity of the pension environment. Such a model will be useful in designing pension systems for future generations in both the United States and other countries, and in understanding more fully the economic role of different types of retirement benefit plans.

The first section of this paper discusses the DB and DC pension models that were developed here, while the second section describes the rest of the economy. The third section describes the way in which the model was solved; the fourth section presents some results; and the fifth section outlines possible model extensions and a conclusion.

2. Defined Benefit and Defined Contribution Pension Plans

Defined Benefit Plan

The worker is assumed to contribute to a defined benefit pension plan over his working life and to receive benefits from the plan when he retires. The date of retirement, and labor supply in general, is assumed to be exogenous, following most of the consumption-savings literature.[2]

The defined benefit pension benefit is assumed to be a constant real proportion κ of final wages for each year of service. The proportion is assumed to be chosen endogenously by the individual when offered the choice to join the DB plan, which may be at the beginning of working life, or later:

$$P_{t+i} = \kappa t \tilde{W}_t \qquad \forall \;\; i > 0 \tag{1}$$

This is a simplified version of a typical DB benefit formula as discussed by Mitchell (2000).[3]

Consistent with theoretical models of the labor market (see, for example, Bulow, 1981 and Bulow and Scholes, 1983) it is assumed that workers pay for defined benefit pensions in the form of lower cash wages. When the individual joins the plan, the expected discounted present value of pension contributions is chosen to equal his expected discounted present value of pension benefits. This makes the simplifying

[2] As discussed by Bodie, Merton and Samuelson (1992) this assumption may have implications for optimal investment and pension strategies.

[3] Most DB plans pay benefits based on an average of the last few years of service, rather than on just the last year, as we assume here. In addition, many DB plans further reduce the variability of the benefit by excluding highly variable compensation (such as sales commissions) from the wages used in the benefit formula. These complications are ignored in this paper for simplicity.

assumption that there are no *ex ante* cross subsidies in the pension plan, even though these are not unusual in defined benefit pension schemes.[4]

Two further issues must be considered when determining contributions to defined benefit pensions plans. The first is whether contributions are increasing, level, or decreasing over time as a proportion of wages. Bodie *et al.* (1988) model contributions as an increasing proportion of wages in order to reflect their view that the value of the corporate pension promise is the termination benefit offered under the plan rules. Nevertheless, this view of DB pensions implies steeply declining cash wages near retirement, and temporary dips in cash wages when an individual's pension vests. This is rejected empirically by Kotlikoff and Wise (1985), Pesando (1985), and others.

An extensive literature on implicit labor contracts and DB pension plans, summarized in Ippolito (1985, 1997) presents the DB pension contract as a long-term implicit contract. The terms of the contract are that the pension plan sponsor promises not to terminate the plan, in exchange for requiring plan members to pay contributions that are higher than the level required to finance the termination benefit. The purpose of these higher contributions (called the 'pension bond' in the literature) is to attract lower discounting and longer tenure workers to the firm. This is in the interests of the firm because longer tenure workers have a greater incentive to accrue firm-specific human capital. These arguments are summarized in Lazear (1979, 1982). By encouraging higher earlier pension contributions, the presence of implicit labor contracts in DB pension plans would tend to imply a more level contribution rate than that assumed by Bodie *et al.* (1988).

For this reason, and for theoretical simplicity, we model the level of contributions as a constant fraction of wages:

$$P_j = -c\tilde{W}_j \qquad \forall \quad 0 < j \le t \tag{2}$$

[4] An example of a common *ex ante* subsidy is that between men and women. Women have lower mortality and hence may earn more valuable benefits. The true extend of cross subsidies is often difficult to measure. See Gustman and Steinmeier (2000) for an analysis of redistribution in Social Security.

Note that the sign in the equation is negative to indicate that these contributions are paid out of wages; benefits have a positive sign. The assumption of a level contribution schedule probably overestimates the extent to which DB pension plans expose members to wage risk. This is because with an increasing contribution schedule, more contributions are made later when more is known about the final level of wages and hence benefits. Thus, contributions and benefits are more likely to be closer together when the contribution schedule is increasing than when the contribution schedule is flat or decreasing.

The level of contributions as a proportion of wages in this model is given by c. In order for the pension to be fully self-financing in expected value, the value of c must depend on the level of investment return the company is willing to guarantee the individual, in exchange for assuming the risk of fluctuating wages. There is a wide literature on this (see Babbell *et al.*, 2002, for a summary), that discusses the risks that firms assume by providing defined-benefit-type guarantees to employees, and the accuracy with which these risks are reflected on company balance sheets. To sidestep this controversy, here we examine defined benefit pension plans in the situation where various levels of 'guarantee' are provided by the corporation. A guarantee of the risk-free rate has the effect of assuming that the employer can diversify wage fluctuations away fully, or that the firm's aggregate real wage bill is uncorrelated with the risky asset. If the firm's wage bill is positively correlated with the risky asset, a higher rate of guarantee can be safely granted.[5]

It should be noted that one of the advantages of DB plans from the viewpoint of the corporate sponsor is that DB plans afford it greater control over employees' retirement behavior. Presumably, this has some value to the employer, implying that some guarantee above the risk-free rate is not unreasonable. Dorsey *et al.* (1998) state that employees would also not accept this restriction on their retirement options unless the firm could pay higher cash wages to compensate, possibly due to higher productivity of

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[5] A more complete analysis, beyond the scope of this paper, could examine the correlation between the aggregate wage bill of a corporation and various risk factors identified in the empirical finance literature (see, for example, Fama and French, 1996).

pension-covered workers (Dorsey *et al.*, 1998 present some evidence on this), the lower turnover of workers in pension-covered jobs (Gustman and Steinmeier, 1995; Allen, Clark and McDermed, 1993), or the sorting effect of pensions (Ippolito, 1997). From the point of view of the employee, this could appear as though the firm were discounting pension benefits at a higher rate than the risk-free rate. This effect is shown in a stylized way in Figure 1. Bodie *et al*. (1988) assumed that the corporate sponsor could fully diversify the impact of wage fluctuations, and that pensions had no productivity, tenure or sorting effects, implying that DB pensions and contributions should be discounted at the risk-free rate.

This paper will use the risk-free rate as a base and will examine the impact on individual portfolio choice if the company is able to provide a slightly larger guarantee, or if the presence of a pension increases worker productivity and hence wages.

It will be assumed that the corporation will not default on its promises, although the analysis can easily be modified to include partial or full default. In the United States, the Pension Benefit Guaranty Corporation provides a federal guarantee for most of the liabilities of defined benefit plans (Ippolito, 1989).

In this model, the corporation pools the pension contributions of individual workers (eliminating wage fluctuations) and uses these to purchase longer-dated risk-free bonds that will pay pension benefits. It is impossible for individuals to do this as they cannot hedge away their wage risk due to incomplete markets. If interest rates were variable, the corporation could issue bonds at time 0, guaranteed by the future pension contributions of its workers, and use the proceeds to purchase much longer-dated bonds to pay benefits. The corporation could redeem its own debt with the employee pension contributions as they fell due.

Defined Contribution Plan

In this paper, we do not model the DC plan assets separately from the other assets of the individual. The DC plan forms part of the assets of the individual and can be invested in stocks and bonds with the worker's other assets. The individual is also free to spend down assets if so desired.

There are several reasons for this. The first is that for simplicity we have not modeled tax incentives for pension saving, or, stated more accurately, tax disincentives for saving outside pension plans. An accurate specification of the tax code would involve complications that are beyond the scope of this paper. In addition, while there is evidence that DB plans pay a slightly higher rate of tax than DC plans (see Ippolito, 1989), both types of plan are largely tax-exempt and thus the effects of introducing taxation would be similar for both plan types. If taxation were introduced, some form of rationing of the tax privilege would need to be modeled, adding further complications. The second reason is that if DC plan assets were segregated from other assets, in this model the optimal level of DC plan would be zero. This is because DC plan assets would be invested in the same assets as non DC plan assets, but unavailable for consumption before retirement.[6] Finally, very few DC plans in the US compel members to annuitize their assets at retirement and less than 25% offer any life annuity option at retirement at all (Mitchell, 1999). Many (especially 401(k) plans) permit members to borrow against accumulated assets. These facts reduce the relevance of segregating the assets of the DC plan.

To prevent the desire of individuals for annuities from biasing the estimated desirability of DB plans upwards, workers are permitted to access the annuity market when they retire. However, unlike DB plan annuities, whose benefit level is chosen when workers join the DB plan, the level of the private annuity can be chosen at retirement. To allow for adverse selection, (see Finkelstein and Poterba, 2002 and Mitchell *et al.*, 1999 for

[6] As DB plans provide access to a security not available in the markets (notably, wage-indexed claims), the same result does not hold for DB plans.

evidence on the presence of adverse selection in annuities markets) the private annuity is not priced neutrally. Letting Y be the amount of income the individual obtains from the annuity, and the price loading for adverse selection in the private annuity market be λ, then the individual must pay:

$$B_{t+1} = Y(1+\lambda)\sum_{i=0}^{s-1} \frac{\pi_{t+1+i}}{\pi_{t+1}} (1+R)^{-i} . \tag{3}$$

R is the rate at which annuity payments are discounted (assumed to be the risk-free rate as an individual's mortality is uncorrelated with the risky asset) and the assumed probability the individual is alive at time i conditional on being alive at time 0 is given by π_i.

3. Model description

A utility maximizing individual works for t periods and then is retired for s periods. As stated above, labor supply is assumed to be exogenous. At the beginning of each period, the individual earns a real risky wage W_j, consumes C_j, and has assets on hand of A_j. Between period $j-1$ and period j, the individual earns a return on assets of R_j. All variables with subscript j are revealed at time j. The model structure is summarised in Table 1.

Table 1 here.

Consumer's wages

A variety of different models of income uncertainty is used in the stochastic income literature (see Pemberton, 1997). There are three main features to all of these models: wages are assumed to be exogenous, to persist over time, and to fluctuate around a permanent income level which may shift over an individual's working life. To model these facts, it is conventional to describe the level of the wage, $\tilde{W}_j$ (earned at the beginning of each period), as:

$$\tilde{W}_j = \exp(\tilde{w}_j)\,, \tag{4}$$

where $\tilde{w}_j$ consists of three components.

The first is a deterministic function that represents the unconditional expected value of wages at time *j*, an individual's permanent income, which is often modeled as a function of demographic variables such as the individual's age, educational achievement and whether or not the individual is head of a household. The second is a permanent error to model shifts in the level of permanent income over an individual's working life. For convenience, this permanent error is assumed to follow an AR(1) process that is either a unit-root process or close to a unit-root process to capture the persistence of wages over time. To model fluctuation around permanent income, there is also assumed to be a transient error that, for convenience, is assumed to be normally distributed with mean zero.[7] This process may be described by the following equations:

$$\tilde{w}_j = f(j,\mathbf{Z}) + \tilde{v}_j + \tilde{\varepsilon}_j \quad \forall\ 1 \le j \le t \tag{5}$$

$$\tilde{v}_j = \theta \tilde{v}_{j-1} + \tilde{\xi}_j \ \ \forall\, 1 \le j \le t \tag{6}$$

$$\tilde{\varepsilon}_j \sim \mathbf{N}(-\tfrac{1}{2}\sigma_\varepsilon^2, \sigma_\varepsilon^2) \tag{7}$$

$$\begin{pmatrix} \tilde{\xi}_j \\ \tilde{\eta}_j \end{pmatrix} \sim \mathbf{N}\left(\begin{pmatrix} -\tfrac{1}{2}\sigma_\xi^2 \\ -\tfrac{1}{2}\sigma_\eta^2 \end{pmatrix}, \begin{pmatrix} \sigma_\xi^2 & \rho\sigma_\xi\sigma_\eta \\ \rho\sigma_\xi\sigma_\eta & \sigma_\eta^2 \end{pmatrix} \right) \tag{8}$$

where $f(j,\mathbf{Z})$ is the individual's permanent income which may depend on a vector of demographic variables **Z**; $\tilde{v}_j$ is the permanent error; and $\tilde{\varepsilon}_j$ the temporary error. The term θ controls the degree of persistence of the permanent income shocks, and $\tilde{\eta}_j$ is the stock

[7] Some authors model a degree of persistence in the transient error as well (see for instance Zeldes, 1989 and MaCurdy, 1982), which is ignored here for convenience.

return at time j (possibly correlated with the permanent error in wages) to be explained below.

The adjustments to the means of $\tilde{v}_j, \tilde{\varepsilon}_j$, and $\tilde{\eta}_j$ of half their variances ensure that:

$$E_0[W_j] = \exp(f(j, \mathbf{Z})) \, . \tag{9}$$

Consumer's Assets

At the beginning of each period j the consumer is assumed to have a stock of assets A_j. He can choose what portion α_j of this to invest in the risky asset, and what portion $1-\alpha_j$ to invest in the risk free asset. A_j is constrained to be greater than 0, implying that the individual cannot borrow against his defined benefit pension plan, and the individual is not permitted to short either the risky or the risk-free asset, implying that $0 \le \alpha_j \le 1$.

The instantaneous return on the risk-free asset in all time periods is given by R.

The instantaneous return on the risky asset in time j is given by $\mu + \tilde{\eta}_j$, where the joint distribution of $\tilde{\eta}_j$ and $\tilde{\xi}_j$ is given in equation (8).[8]

The formula above implies that there may be a correlation between the permanent portion of an individual's income shocks and the returns on the risky asset.

The return on an individual's portfolio is given by:

[8] Campbell and Viciera (1999) have modeled the case where the equity risk premium is time-variant and there is some predictability in asset returns. The issue of time-varying returns may affect pension choice in that individuals may be more likely to choose DB plans when DC plans are expected to produce lower returns. However, as a first assumption, this model will adopt the approach of assuming a constant equity risk premium.

$$\tilde{R}_{j+1} = \alpha_j[\exp(R + \mu + \tilde{\eta}_{j+1}) - \exp(R)] + \exp(R). \tag{10}$$

This equation assumes that the individual balances his portfolio only at the beginning of the year, rather than continuously. The standard adjustment to the mean of $\tilde{\eta}_{j+1}$ ensures that:

$$E[\tilde{R}_{j+1}] = \alpha_j \exp(R + \mu) + (1 - \alpha_j)\exp(R) \tag{11}$$

Assuming that the individual consumes C_j at time j, the transition equation for the assets of the individual is:

$$A_{j+1} = \tilde{R}_{j+1}(A_j - C_j + \tilde{W}_j 1_{j \le t} + P_j) \quad > \quad 0 \;\; \forall \;\; j < t \tag{12}$$

where

$$1_{j \le t} = \begin{cases} 1 \;\forall\; j \le t \\ 0 \text{ otherwise} \end{cases}. \tag{13}$$

Equation (12) assumes that after cash flows in period j, the individual has assets (Deaton's (1991) 'cash-on-hand') that equal the assets in period j before cash flows, less consumption in period j plus income in period j. If the individual is working, income equals wages less pension contributions. These assets, plus investment income received on them over the year, comprise the individual's assets or cash-on-hand before cash flows in period $j+1$.

Just after retirement, the individual has the option to spend B_{t+1} on buying an annuity in the private market, as discussed in the section on DC plans above. The definition of B_{t+1} is given in equation (3). Then the asset transfer equation between times t and t+1 is given by:

$$A_{t+1} = \tilde{R}_{t+1}(A_t - C_t + \tilde{W}_t + P_t) - B_{t+1} \quad > \quad 0 \tag{14}$$

The constraint on A_{t+1} implies a constraint on B_{t+1}.

During retirement the individual receives and income equal to his DB pension plus his annuity income.

This, income in retirement is given by:

$$Y + \kappa t \tilde{W}_t = Y + P_{t+i} \tag{15}$$

The asset transfer equation for assets during retirement is then given by:

$$A_{t+i+1} = \tilde{R}_{t+i+1}(A_{t+i} - C_{t+i} + P_{t+i} + Y) \quad > \quad 0 \quad \forall\, i > 0\,. \tag{16}$$

Decision Structure

At time 0, the individual is assumed to maximize a time-separable expected utility function of the following form:

$$\max_{\kappa} \max_{\{Y, C_i, \alpha_i\}} E_0 \sum_{i=1}^{t+s} \beta^i \pi_i u(C_i) \tag{17}$$

where:

$$u(C) = \frac{C^{1-\gamma}}{1-\gamma}\,,\ \gamma \geq 1\,, \tag{18}$$

and π_i is the assumed probability the individual is alive at time *i* conditional on being alive at time 0. This implies that at time 0, the individual chooses the level of κ (the defined benefit plan accrual rate), C_0 (his consumption at time 0) and α_0 (the asset mix of his portfolio at time 0). At each time thereafter, the individual will choose only C_j (his consumption at time *j*) and α_j (the asset mix of his portfolio) conditional on the chosen value of κ. At time *t*, the individual chooses the value *Y* (the amount of the private market annuity).

The parameters of the model are thus: $\mathbf{Z}, \theta, \sigma_\varepsilon^2, \sigma_\xi^2, \sigma_\eta^2, \sigma_{\xi\eta}, \rho, R, \mu, \gamma, \beta, \pi, f, \lambda$ where the other parameters may depend on elements of **Z**.

Some elements of this model (the wage process, the asset process and the choice of utility function) follow Campbell *et al.* (2000). The model in this paper is also closely related to the 'buffer-stock saving' model of Carroll (1992). An issue to consider when using lifetime consumption models that include a risky asset is that most models of this type fail to match both consumption-savings profiles and observed asset mixes. This is the case even when high fixed costs of participating in the equity market are assumed and risk aversion is very high. The reason for this is the historically high equity risk premium, examined by Mehra and Prescott (1985) and many others. This paper, like many others in this literature, will not attempt to resolve this paradox, which is heightened by the presence of labor income driving individuals to hold stocks.

4. Model Solution

This section describes the techniques used to solve the model, similar to those adopted by Carroll (1997a).

Derivation of Euler Equations

In the last period, the individual consumes all assets. Thus,

$$C_{t+s} = A_{t+s} + \kappa t W_{t+s} + Y = A_{t+s} + P_{t+s} + Y \tag{19}$$

Let $X = P_{t+1} + Y$, the worker's (constant) income in retirement. Then, in retirement, the value function is a function of two state variables, X and A_{t+i}. In the last period:

$$V_{t+s}(A_{t+s}, X) = u(A_{t+s} + X) \tag{20}$$

In other periods, the individual must balance future consumption and present consumption. Each year after time t, including at time $t+1$, the individual must solve:

$$\begin{aligned} V_j(A_j, X) &= \max_{\{C_j,\alpha_j\}} u(C_j) + \beta \frac{\pi_{j+1}}{\pi_j} E_j V_{j+1}(\tilde{R}_{j+1}(A_j - C_j + X), X) \\ &\text{s.t. } \; C_j \le A_j + X \end{aligned} \tag{21}$$

At time $t+1$, the year after retirement, the individual must decide how much of his wealth to annuitize by solving the following problem:

$$\begin{aligned} V_{t+1}(A_{t+1}, X) &= \max_{\{Y\}} V_{t+1}(A_{t+1} - B_{t+1}, P_{t+1} + Y) \\ &\text{s.t. } \; B_{t+1} = Y(1+\lambda) \sum_{i=0}^{s-1} \frac{\pi_{t+1+i}}{\pi_{t+1}} (1+R)^{-i} \end{aligned} \tag{22}$$

At time t, the individual must solve the following problem:

$$\begin{aligned} V_j(A_t, \varepsilon_t, v_t) &= \max_{\{C_t,\alpha_t\}} u(C_t) + \beta \frac{\pi_{t+1}}{\pi_t} E_t V_{t+1}(\tilde{R}_{t+1}(A_t - C_t + W_t + P_t), X) \\ &\text{s.t. } \; C_t \le A_t + W_t + P_t \\ &\qquad X = t\kappa W_t + Y \, , \end{aligned} \tag{23}$$

where Y is the optimal value chosen in equation (22) for each level of DB pension and wealth.

Before time t, the value function is a function of three state variables, A_i, ε_i and v_i. At these times, the worker must solve the following problem:

$$V_j(A_j,\varepsilon_j,v_j)=\max_{\{C_j,\alpha_j\}} u(C_j)+\beta\frac{\pi_{j+1}}{\pi_j}E_jV_{j+1}(\tilde{R}_{j+1}(A_j-C_j+W_j+P_j),\tilde{\varepsilon}_{j+1},\tilde{v}_{j+1}) \quad (24)$$

$$\text{s.t.}\quad C_j\le A_j+W_j+P_j$$

To avoid repetition, only the derivations of results for the more complex pre-retirement time periods will be shown. Results at and after retirement can easily be obtained by following the same procedure.

Ignoring constraints on consumption, the F.O.C. for consumption is:

$$u'(\hat{C}_j)=\beta\frac{\pi_{j+1}}{\pi_j}E_j\tilde{R}_{j+1}V'_{j+1}(\tilde{R}_{j+1}(A_j-\hat{C}_j+W_j+P_j),\tilde{\varepsilon}_{j+1},\tilde{v}_{j+1}), \quad (25)$$

where "'" denotes differentiation w.r.t. the first argument of the function to which it is applied.

The envelope theorem says:

$$V'_j(A_j,\varepsilon_j,v_j)=\beta\frac{\pi_{j+1}}{\pi_j}E_j[\tilde{R}_{j+1}V'_{j+1}(\tilde{R}_{j+1}(A_j-\hat{C}_j+W_j+P_j),\tilde{\varepsilon}_{j+1},\tilde{v}_{j+1})], \quad (26)$$

which implies that:

$$u^{'}(\hat{C}_j) = V_j^{'}(A_j, \varepsilon_j, v_j)\text{, or that } u^{'}(\hat{C}_{j+1}) = V_{j+1}^{'}(A_{j+1}, \varepsilon_{j+1}, v_{j+1})\,. \tag{27}$$

Combining (25) and (27) gives the Euler Equation for consumption:

$$u^{'}(\hat{C}_j) = \beta \frac{\pi_{j+1}}{\pi_j} E_j[\tilde{R}_{j+1} u^{'}(\hat{C}_{j+1})]\,. \tag{28}$$

Similar arguments show that equation (28) applies at all other time periods except the last period.

Again ignoring constraints, the F.O.C for asset allocation is:

$$0 = E_j[(\exp(R + \mu + \tilde{\eta}_{j+1}) - \exp(R))V_{j+1}^{'}(\hat{\tilde{R}}_{j+1}(A_j - C_j + W_j + P_j), \tilde{\varepsilon}_{j+1}, \tilde{v}_{j+1})] \tag{29}$$

or,

$$0 = E_j[(\exp(\mu + \tilde{\eta}_{j+1}) - 1)V_{j+1}^{'}(\hat{\tilde{R}}_{j+1}(A_j - C_j + W_j 1_{j \le t} + P_j + Y 1_{j > t}), \tilde{\varepsilon}_{j+1}, \tilde{v}_{j+1})]\,. \tag{30}$$

The two F.O.C.'s (30) and (28) need to be solved simultaneously for $\hat{\alpha}_j$ and $\hat{C}_j$.

In order to do this, define a function:

$$\Omega_j(s_j, \alpha_j, \varepsilon_j, v_j) = E_j[V_{j+1}(\tilde{R}_{j+1} s_j, \alpha_j, \tilde{\varepsilon}_{j+1}, \tilde{v}_{j+1})] \text{ where } s_j = A_j + P_j + W_j \tag{31}$$

Hence,

$$\Omega_j^s(s_j, \alpha_j, \varepsilon_j, v_j) = E_j[\tilde{R}_{j+1} V_{j+1}^{'}(\tilde{R}_{j+1} s_j, \alpha_j, \tilde{\varepsilon}_{j+1}, \tilde{v}_{j+1})], \tag{32}$$

and

$$\Omega_j^{\alpha}(s_j,\alpha_j,\varepsilon_j,v_j)=E_j[(\exp(R+\mu+\tilde{\eta}_{j+1})-\exp(R))V_{j+1}^{'}(\tilde{R}_{j+1}s_j,\alpha_j,\tilde{\varepsilon}_{j+1},\tilde{v}_{j+1})]\,.(33)$$

Equations (32) and (33) in conjunction with the envelope theorem argument used above, imply that

$$\Omega_j^{s}(s_j,\alpha_j,\varepsilon_j,v_j)=E_j[\tilde{R}_{j+1}u^{'}(\hat{C}_{j+1}[\tilde{R}_{j+1}s_j,\alpha_j,\tilde{\varepsilon}_{j+1},\tilde{v}_{j+1}])]\,, \tag{34}$$

and

$$\Omega_j^{\alpha}(s_j,\alpha_j,\varepsilon_j,v_j)=E_j[(\exp(R+\mu+\tilde{\eta}_{j+1})-\exp(R))u^{'}(\hat{C}_{j+1}[\tilde{R}_{j+1}s_j,\alpha_j,\tilde{\varepsilon}_{j+1},\tilde{v}_{j+1}])] \tag{35}$$

Hence, the F.O.C.'s can be rewritten as

$$u^{'}(\hat{C}_j)=\beta\frac{\pi_{j+1}}{\pi_j}\Omega_j^{s}(s_j-\hat{C}_j,\hat{\alpha}_j,\varepsilon_j,v_j)\,, \tag{36}$$

and

$$0=\Omega_j^{\alpha}(s_j-\hat{C}_j,\hat{\alpha}_j,\varepsilon_j,v_j)\,. \tag{37}$$

These four equations (34, 35, 36 and 37), along with the solution for the first retirement period, can be used recursively to obtain the optimal consumption rule for all time periods except time *t*, the date of retirement. The first retirement period solution is obtained by recursively solving four similar equations, starting at the last period, and then solving for the optimal annuitisation strategy. While technically the individual solves the annuitisation, consumption and asset allocation decisions in this time period simultaneously, these decisions can be solved as sequential decisions.

Note that the constraint $A_j \geq 0$ is binding when

$$u^{'}(A_j + P_j + W_j) = \beta \Omega_j^s(0, \hat{\alpha}_j, \varepsilon_j, v_j)\,, \tag{38}$$

or when

$$A_j = u^{'}[\beta \Omega_j^s(0, \hat{\alpha}_j, \varepsilon_j, v_j)]^{-1} - P_j - W_j \text{ and } 0 = \Omega_j^{\alpha}(0, \hat{\alpha}_j, \varepsilon_j, v_j)\,. \tag{39}$$

The constraint $0 \leq \alpha_j \leq 1$ is binding when the unconstrained solution lies outside the range [0,1].

Algorithms

The Gaussian random variables were discretised by following the approach of Carroll (1992). His technique was extended to permit multiple, correlated random variables by discretising the variables in sequence and using conditional distributions where appropriate.

The Euler equations were solved numerically using a Newton-Raphson method. The value functions and the derivative of the value functions were approximately linearized by the use of appropriate power transformations. This reduced the number of sample points in the grid that were required to estimate the value functions accurately. The technique was effective provided that the values at which the constraints bind were included in the grid of values for which the function $\Omega_j(s_j, \alpha_j, \varepsilon_j, v_j)$ (defined in equation 31) was estimated. The interpolation over the values of the state variable v_i was performed using exponential interpolation to reflect the fact that its value affects income in an exponential way, while ε_i was modeled as a lognormal, multiplicative random variable of income to avoid this problem.

The asset mix was chosen first for each level of consumption and each combination of the state variables. Then, the optimal level of consumption was chosen for each combination of the state variables. This technique used the principle of dynamic programming applied over different choice variables rather than over different time periods. It offered improved computation time and reliability over a simulataneous solution of the Euler equations. The same approach was used to determine the optimal annuitization strategy at retirement.

The optimal value of κ was chosen by means of an optimization algorithm over the values of $V_0(A_0, \nu_0, \varepsilon_0)$, corresponding to different levels of income and wealth. The optimization procedure used either cubic spline interpolation or linear bisection of the estimated value function derivatives, conditional on whether the cubic spline interpolation produced results that lay inside the bisection interval.

The solution was implemented in a combination of Matlab and C++. The problem of retirement was solved only once (using the technique of state variable reduction) and stored for each set of parameters, allowing pre-retirement runs to be performed fairly rapidly.

5. Assumptions and Results

The assumptions used by the model are laid out in Table 2.

Table 2 here.

The model assumes an equity risk premium of 4% p.a., lower than historical market values would suggest. The standard deviation of equity returns is in line with historical values of 15.7% p.a. The benchmark risk aversion used is 5, slightly higher than many authors have used. These three assumptions are chosen to ensure that the benchmark proportion of wealth invested in the risky asset is not too far from observed values (see Heaton and Lucas, 2000). The presence of labor income (and pension income in retirement) implies that much higher equity positions will be observed in this model than the benchmark.[9]

Many authors (Carroll, 1992, Carroll and Samwick, 1997 and 1998, Heaton and Lucas, 2000, and Campbell *et al.*, 2000) have examined the PSID and obtained estimates of earnings volatility and wage profiles. There is thus little value added by performing another analysis of the same dataset in this context. For the estimates of the variance of permanent income shocks, I have decided to use the estimates of Campbell *et al.* (1999), which is approximately 30% higher than the estimates used by other researchers. For the temporary shock variance, I use a value higher than that used by Carroll (1992) and Heaton and Lucas (2000), but a lower estimate than obtained by Campbell *et al.* (2000). It should be noted that measurement error can have a large effect on estimates of temporary variance in earnings. Most DB pension plans use an average of the last few years' income to determine the pension benefit, rather than the last year as we have assumed here, which will average out wage fluctuations. These facts allow a lower estimate of the temporary variance of income shocks to be used in this context.

Figure 2 here.

The shape of the wage profile seems to have a small impact on the results, and hence the profile obtained by Campbell *et al.* (2000) from the PSID for college-educated students was chosen. This wage profile is shown in Figure 2. Some studies that provide aggregate estimates for the covariance of permanent wage shocks and risky asset returns include Heaton and Lucas (1997), who estimate this correlation at the individual level and find that most workers have a correlation of between -0.1 and 0.2 between wage shocks and the risky asset. They do not state the sampling distribution of their estimated

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[9] A standard benchmark used is Merton (1969)'s result that, in the absence of labor income, the proportion of wealth invested in the risky asset equals $\mu/(\sigma_\eta^2\gamma)$ in the notation of this model. For these parameter values, this is approximately 1/3.

correlation. Davis and Willen (2000) estimate this correlation for different professions and find typically small positive values except for individuals who are self employed. For this paper, we are thus justified in assuming a small positive value for this parameter, following Campbell *et al.* (2000).

Figure 4 here.

Figure 4 shows the solution of this model for a worker who enters the labor force at age 23 for the assumptions described above. This worker is assumed to contribute nothing to the pension plan, so $\kappa = 0$. The graph shows the worker's annual earnings, asset holdings, consumption, and asset mix between equities and bonds. This figure assumes that all random variables equal their expected values in each time period. As expected, just before retirement, when human capital has been exhausted, the individual's equity holdings are approximately 1/3. The worker's assets fall dramatically at retirement as the annuity is purchased. After retirement, the riskless annuity crowds out all bond holdings. In early years, the individual saves very little, but starts to save for retirement from around age 30.

The optimal values of κ are presented in two separate tables. Table 3 shows the impact of an individual's time horizon and initial wealth on the choice to participate in a DB pension scheme, and is illustrated graphically in Figure 4. Table 4 shows the sensitivity of the optimal DB choice to various assumptions.

Table 3 here.

The results show that at early ages, DB pension plans are not a desirable vehicle for retirement saving. Even for relatively wealthy individuals, shown in the third column of Table 3, the optimal DB replacement rate for individuals younger than age 45 is 0. There are three reasons for this. The first is that the return on the DB benefit (the risk free rate plus the 10% adverse selection cost, spread over the number of years to retirement) is too low relative to its variance. The variance equals the variance of the final year's earnings conditional on the state variables at the time the decision is made. For an individual aged 25, the variance of earnings over 40 years is simply too large to justify returns that are close to the risk-free rate. The second reason is that when he is young, the worker has most of his wealth invested in human capital, which is highly correlated with the final benefit of the DB plan. For younger individuals, equities provide an opportunity to diversify away from human capital. As the individual ages, human capital is converted into financial wealth, and his wage uncertainty is resolved. This implies that the diversification value of DB pension plans increases and the variability of the final benefit decreases. Hence, we would expect that, as shown in Table 3, (illustrated in Figure 4) as the individual nears retirement, DB plans become progressively more attractive. The final reason causing DB plans to become more attractive as workers near retirement is that the annual rate of return on the DB plan increases. This is because the fixed benefit of participating in the group annuity market through the DB plan as opposed to in the (unfair) private market is spread over fewer years.

The results in Table 3 show that the advantages of DB plans for older workers are very large. For instance, a 57 year-old individual with financial wealth equal to 5 times his current earnings, has an optimal DB replacement rate of 35% - a huge number considering that he only has 8 working years to contribute to the pension plan. For older individuals, the benefits of DB plans are so substantial that the model predicts that workers would invest most or all of their earnings in these plans if given the opportunity. This is illustrated by the optimal DB replacement rates marked by asterisks, which indicate that workers are constrained from investing more than their entire earnings in their DB plan. While this is admittedly unrealistic, it illustrates the value that workers near retirement place on DB plans: DB plans diversify the asset portfolio out of financial markets and provide cheap access to the market for life annuities.

Table 3 also shows how the optimal DB pension plan replacement rate varies by the initial financial wealth of the individual at the time the option is given to participate in the DB plan. The higher his initial wealth relative to income, the greater the proportion of total wealth (including human capital) that is invested in financial capital and hence the greater the diversification benefit of investing in the DB plan. If wealth is higher, the individual can also sacrifice a greater proportion of earnings into the DB plan without adversely affecting consumption.

Figure 5 here.

Figure 6 here.

Figures 5 and 6 show the consumption-earnings profiles from age 56 for the base case of the model when the worker chooses no DB pension plan and when the worker chooses the optimum level of the DB pension plan, respectively. Again, both these figures assume that all random variables equal their expected values.

Figure 5 shows that the individual with no DB plan accumulates around $300 000 just before retirement, and has approximately 1/3 of this invested in the risky asset, as expected. At retirement, the individual purchases an annuity on the private market that gives him a replacement rate of about 45%. After retirement, this annuity crowds out his bond holdings and all assets are held in the risky asset thereafter.

Figure 6 shows the same profiles for an individual who chooses the optimum DB participation rate at that age. This figure shows two lines for income – one before the pension contributions are deducted and the pension annuity is received, and one after this is done. Several features of this graph stand out in comparison with Figure 4. Firstly, this individual has only about $180 000 in financial assets before retirement because the DB plan has crowded out other savings. This crowd-out comes disproportionately from bonds, which now represent only around 40% of the individual's portfolio just before retirement. At retirement, the worker buys an annuity on the private market with a replacement rate of only 20%. The optimal DB replacement rate for the base case is 28% (read off Table 3), implying that the total replacement rate for this individual is 48%. The reason that this is higher than the individual who did not participate in the DB plan is that the DB annuity is cheaper than the private market annuity. As before, the annuities crowd out all the bond holdings in retirement and this retiree holds only bonds until the end of his life.

Table 4 shows the sensitivity of the optimal DB replacement rate to various parameters in the model. The sensitivity of the model to time to retirement and initial wealth have already been discussed.

Table 4 here.

Table 4 shows that the factors affecting the desirability of DB pension plans most are the annuity loading, the wage variability factor and the DB guarantee.

If the private annuity market loading falls, DB plans become much less attractive. In the case that the private annuity market is actuarially fair, the optimal DB replacement rate is 0. This is because the DB plan pays the risk-free rate but provides a risky benefit and is fully dominated by the risk-free asset. We would expect DB pension plans to be more common in environments where private annuity markets are inefficient.

As wage variability increases, the optimal DB replacement rate falls. This is because the riskiness of the DB benefit increases as wage variability increases. We expect DB plans to be more common in industries where wage variability is lower – such as governments and large corporations.

If the DB guarantee increases, the optimal DB replacement rate increases. This is because the expected value of the DB benefit increases while its variance remains unchanged. We therefore expect that DB plans would be more prevalent in industries where the aggregate wage bill is highly correlated with the risky asset or where the employer is willing to pay a premium to control the retirement behavior of employees.

The remaining factors affect the desirability of DB pension plans, but less dramatically.

An increase in risk aversion causes the optimal DB pension plan replacement rate to fall. This produces the surprising result that DB plans are more attractive to individuals who are less risk averse – at least over the range of risk aversion examined here. This conclusion is in line with the conclusion of Bodie *et al.* (1988). The shape of the earnings profile (as altered by the productivity parameter) has a small impact on the optimal level of DB earnings. As earnings growth increases, the optimal DB replacement rate falls slightly, possibly because increased earnings growth reduces saving in general and increases the absolute level of the final benefit. An increase in the equity risk premium causes the optimal DB replacement rate to fall because the attractiveness of the main investment alternative – equities – is increased. The rate of time preference has only a very small impact on the desirability of DB pension plans.

The model shows that under a variety of assumptions, DB plans can provide value to individuals nearing retirement - primarily by providing additional diversification out of the financial markets and cheaper access to annuities markets. For individuals more than about 15-20 years from retirement, however, the value of DB plans is likely to be small due to high wage uncertainty, the low annual return required to compensate for the cheaper access that DB plans provide to annuities markets, and the high proportion of the total wealth of younger workers that is invested in their human capital.

Extensions and conclusion

The model described here could be extended in a number of ways in future research.

Habit formation

A possible advantage of DB plans over DC plans is that they provide individuals who have long working histories a benefit with a stable replacement ratio. This would make DB plans more attractive if individuals have habit-formation-type preferences. This model could easily be extended and results obtained for habit formation utility functions, as Carroll (2000) has done for the buffer-stock savings/consumption model.

Social Security

An important source of retirement income for most people in the US is Social Security, which is a re-indexed career-average defined benefit plan with some redistributive aspects. Baxter (2001) examines Social Security as a financial asset, and she finds surprisingly low correlations between Social Security benefits and final wages. Her work implies that the diversification element of DB plans identified here would still be valuable if Social Security were included. However, because Social Security also provides access to the annuities market, DB plans may well be crowded out by Social Security, as discussed by Mitchell *et al.* (1999). The model presented here could easily be extended by forcing individuals to pay an amount of their pay, sufficient to provide replacement ratios similar to Social Security, into an account that provides them with an annuity at a group rate. The attractiveness of DB pensions could be examined as a function of the replacement rate provided by Social Security.

Annuity demand models

The annuity demand model in this paper is relatively unsophisticated. Other authors (see, for example Mitchell *et al.* (1999)) have modeled much more complicated demand models, including such aspects as bequest motives, medical expenses and housing – all of which may reduce the demand for annuities. The high value of annuities implied by our model may overstate the value of defined benefit plans' provision of access to an actuarially fair annuity. Alternatively, the fact that annuity rates vary considerably over time adds a further dimension of risk to the private annuity market that could easily be modeled.

Mandatory DC plan

Finally, it is important to examine the case of forced saving. In many countries with mandatory DC pension plans (see Piggott (2002)) individuals are forced to save for retirement from an early age. It is probable that the diversification element of DB plans would still be useful for individuals nearing retirement, although the annuitisation advantage of DB plans would no longer hold if annuitisation were mandatory in the DC system. It is also possible that the forced savings in the DC plan would crowd out some of the DB savings later in life. The extension to this model is not difficult to achieve.

Conclusion

This paper has identified conditions under which workers use a defined benefit pension to help bear investment and wage risk. At the levels of wage risk assumed here, wage indexed claims are not a suitable vehicle for retirement savings for young individuals. This is because investment in stocks over the lifecycle is expected to fully dominate a wage-indexed defined benefit plan. But a defined benefit plan does offer a valuable alternative to workers nearing retirement. For such individuals, the risks of a defined benefit plan are likely to be smaller, as most wage variability has already been resolved. Further, the returns are likely to be higher than for younger workers, as the benefit of cheaper access to annuities markets is spread over a smaller number of years. In addition, because individuals have relatively little of their wealth in human capital, and instead hold more financial assets, the diversification of wealth out of financial markets can be extremely valuable at older ages.

Our results also show that DB plans are more valuable if financial wealth is higher relative to income; if wage variability is lower; if access to private annuities markets is expensive and if the equity risk premium is lower.

These results suggest that a retirement policy that encourages firms to allow older workers to join defined benefit plans, or a national wage-linked social security system for older workers will be welfare enhancing. This is particularly true in countries where annuity markets are poorly developed, or where equity markets are risky and provide low expected returns.

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Table 1: Model Schematics

	Working period				Retirement period		
Time	*1*	*2*	...	*t*	*t+1*	...	*t+s*
Income	W_1	W_2	...	W_t	*0*	...	*0*
Wealth	A_1	A_2	...	A_t	A_{t+1}	...	A_{t+s}
Pension	P_1	P_2	...	P_t	P_{t+1}	...	P_{t+s}
Consumption	C_1	C_2	...	C_t	C_{t+1}	...	C_{t+s}
Risky Asset Return		R_2	...	R_t	R_{t+1}	...	R_{t+s}

Notes to Table 1:

At each time period the individual chooses how much to consume and how much to save from current income. The asset mix of savings (between bonds and equities) can be adjusted each period. Income is stochastic with permanent and temporary errors. Risky asset returns are assumed to be lognormally distributed with a constant mean and variance.

At time 0, the individual chooses whether to join a DB pension plan and can choose the annual accrual rate. The DB pension plan pays a pension benefit equal to the annual accrual rate multiplied by the individual's length of service (*t* in this model) multiplied by the individual's final wage. This pension benefit is paid annually until death. Once this choice is made, the individual cannot opt out of the DB pension plan. The individual pays a level proportion of wages to the pension plan, where the proportion is chosen so that the expected discounted present value of contributions equals the expected discounted present value of benefits at some interest rate. The DB pension plan does not pay a lump sum benefit.

At retirement, the individual can choose to purchase an annuity from private savings on the private market. To model the costs of adverse selection, the annuity is not priced fairly but has a multiplicative loading factor incorporated into the price. This annuity pays a level annual pension for life. The individual can purchase an annuity regardless of whether he participated in the DB plan.

There is no bequest motive, labor supply is assumed to be exogenous and the individual is not permitted to borrow either stocks or bonds.

The individual maximizes:

$$\max_{\kappa} \max_{\{Y, C_i, \alpha_i\}} E_0 \sum_{i=1}^{t+s} \beta^i \pi_i u(C_i)$$

where

$$u(C) = \frac{C^{1-\gamma}}{1-\gamma}, \ \gamma \geq 1, \text{ and}$$

π_i is the assumed probability the individual is alive at time *i* conditional on being alive at time 0.

Table 2: Description of model assumptions.

Assumption	
Risk aversion	5
Time preference	4%
Risk-free Interest rate	2%
Equity risk premium	4%
Equity uncertainty	σ_{η} = 0.157
Permanent Income Profile	Polynomial profile [b]
Income Uncertainty	θ = 1 σ_{ξ} = 0.130 σ_{ε} = 0.121 [c]
Mortality	US Females [d]
Equity / permanent wage error covariance	$\sigma_{\eta\xi}$ = 0.00181
Liquidity Constraints	Directly Imposed
Private Annuity Market	λ = 10% [e]

Notes to Table 2:

[a] No bequest motive is assumed and labor supply is assumed to be exogenous.
[b] The profile was taken from Campbell *et al.* (1999), estimated from the PSID separately for college-educated individuals, following Hubbard, Skinner and Zeldes (1995). The profile is shown in Figure 2.
[c] Many authors have used the PSID to obtain estimates of income variance. The values used here are slightly higher than those used by Hubbard, Skinner and Zeldes (1995), Heaton and Lucas (2000), and Carroll (1992). These results were derived from Campbell *et al.*'s analysis of the PSID. Temporary standard deviations were his results for college-educated individuals, halved to allow for measurement error.
[d] The projected mortality of the cohort of US Females born in 1980, calculated by the Berkeley mortality database with data from the Social Security Administration was used. For information about the tables, go to http://demog.berkeley.edu/wilmoth/mortality.
[e] Mitchell *et al* (1999) estimate adverse selection and loading costs to be around 10% of the cost of annuities.

Table 3: Optimal DB replacement rates by age of option and initial financial wealth:

	Initial cash-on-hand / Initial wages [b]		
Age [a]	2	5	8
42	0	0	0
43	0	0	0
44	0	0	0
45	0	0	0.02
46	0	0.02	0.07
47	0	0.04	0.10
48	0	0.06	0.13
49	0.01	0.09	0.15
50	0.03	0.12	0.18
51	0.05	0.14	0.21
52	0.08	0.17	0.25
53	0.10	0.21	0.28
54	0.13	0.24	0.32
55	0.15	0.28	0.36
56	0.18	0.31	0.40
57	0.21	0.35	0.45
58	0.24	0.39	0.48
59	0.26	0.41	0.42*
60	0.26	0.34*	0.34*

Notes to Table 3:

The table shows the optimal DB replacement rate (*i.e.* the optimal DB pension as a proportion of final salary) for the model described in the text, with assumptions as listed in Table 2. The table assumes the wage profile of a college-educated individual, as shown in Figure 2, with temporary and permanent earning shock standard deviations as shown in Table 2. No high school and high-school educated individuals have quantitatively similar results and hence are not shown. In the notation of the model, the figures listed equal $t\hat{\kappa}$, the years of service multiplied by the optimal annual accrual rate. Figures shown to calculated precision. At earlier ages, optimal DB plan participation equals 0 for all levels of wealth below the largest one listed. An * indicates a solution where the contribution to the DB pension plan equals an individual's entire salary.

[a] This is the age at which the option to join the pension plan is granted – *i.e.* 65-t in the notation of the model.
[b] Cash-on-hand is all wealth not invested in the pension plan and excluding the present value of future wages. Initial wages is the first wage the individual receives.

Table 4: Parameter Sensitivity.

Risk aversion (ρ)		Optimal DB replacement rate [e]
3		0.37
5		0.28
7		0.14
Annuity loading		Optimal DB replacement rate [e]
0%		0.00
5%		0.14
10%		0.28
Productivity growth [a]		Optimal DB replacement rate [e]
0% p.a.		0.28
1% p.a.		0.26
2% p.a.		0.24
Equity risk premium		Optimal DB replacement rate [e]
2% p.a.		0.31
4% p.a.		0.28
6% p.a.		0.23
Rate of time preference		Optimal DB replacement rate [e]
2% p.a.		0.29
4% p.a.		0.28
6% p.a.		0.27
Wage variability factor [b]		Optimal DB replacement rate [e]
0.5		0.56
1		0.28
1.5		0.03
DB Guarantee [d]		Optimal DB replacement rate [e]
0% p.a.		0.28
1% p.a.		0.38
2% p.a.		0.77

Notes to Table 4:

Where an assumption is not stated, its value is as in the base case, described in Table 2. This table uses the wage profile and variability estimates of college-educated students, as adjusted and described in Table 2. Results for high-school educated and no-high school educated individuals are quantitatively similar and hence are not shown. An * indicates a solution where the DB contribution rate equals the individual's entire salary.

[a] This is an annual percentage adjustment made to the wage profile discussed in the notes to Table 2 above to compensate for the fact that the estimation procedure used may not have fully captured the effects of rising productivity due to the short sample period. Base value is 0.

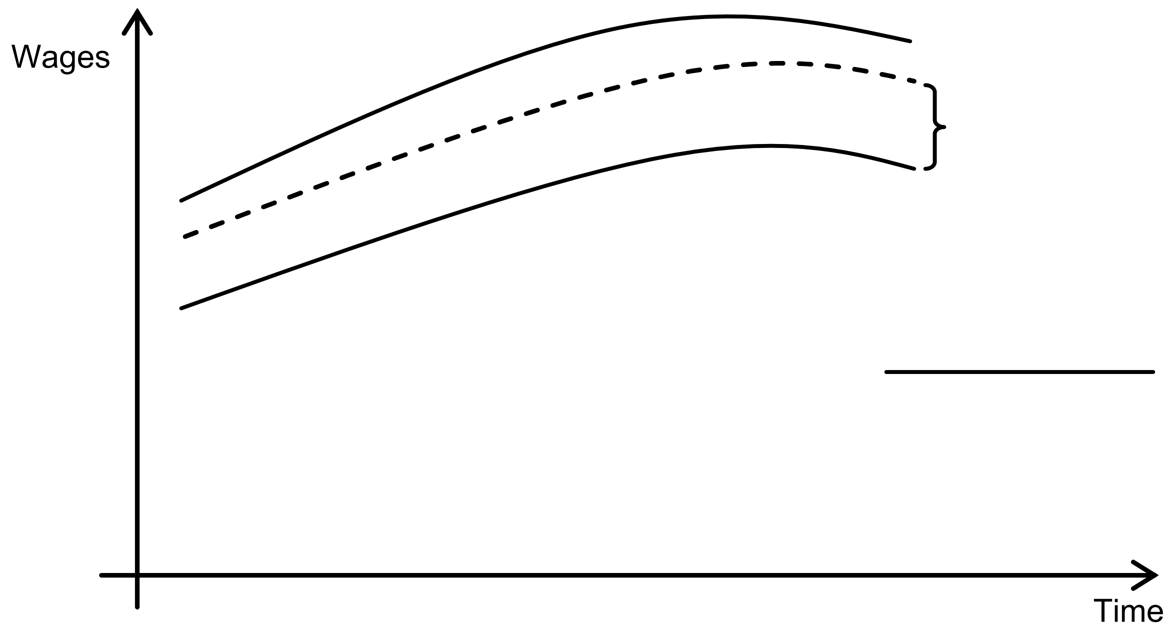
[b] This is a factor by which the temporary and permanent components of wage variability (presented in Table 2) are multiplied to illustrate the effect of variation in wages. Base value is 1.0.

[c] This ratio is described in the notes to Table 3.

[d] This is an amount added to the risk-free rate when calculating the level proportion of wages the individual must contribute to the DB pension plan. Base value is 0.

[e] In the notation of the model, the figures listed equal $t\hat{\kappa}$, the years of service multiplied by the optimal annual accrual rate.

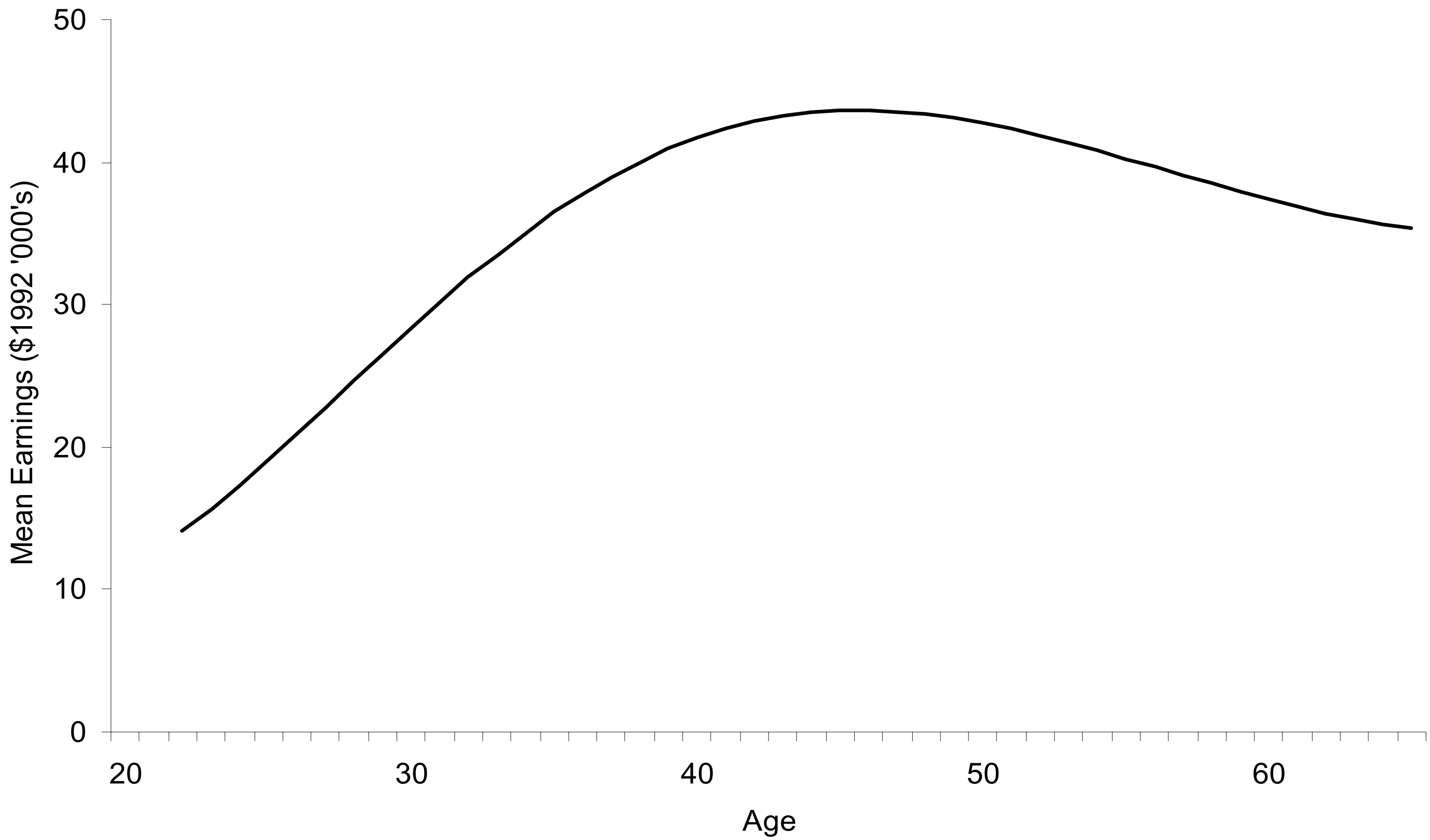
Figure 1: The impact of pension-induced productivity improvements on apparent pension discount rate



Notes to Figure 1:

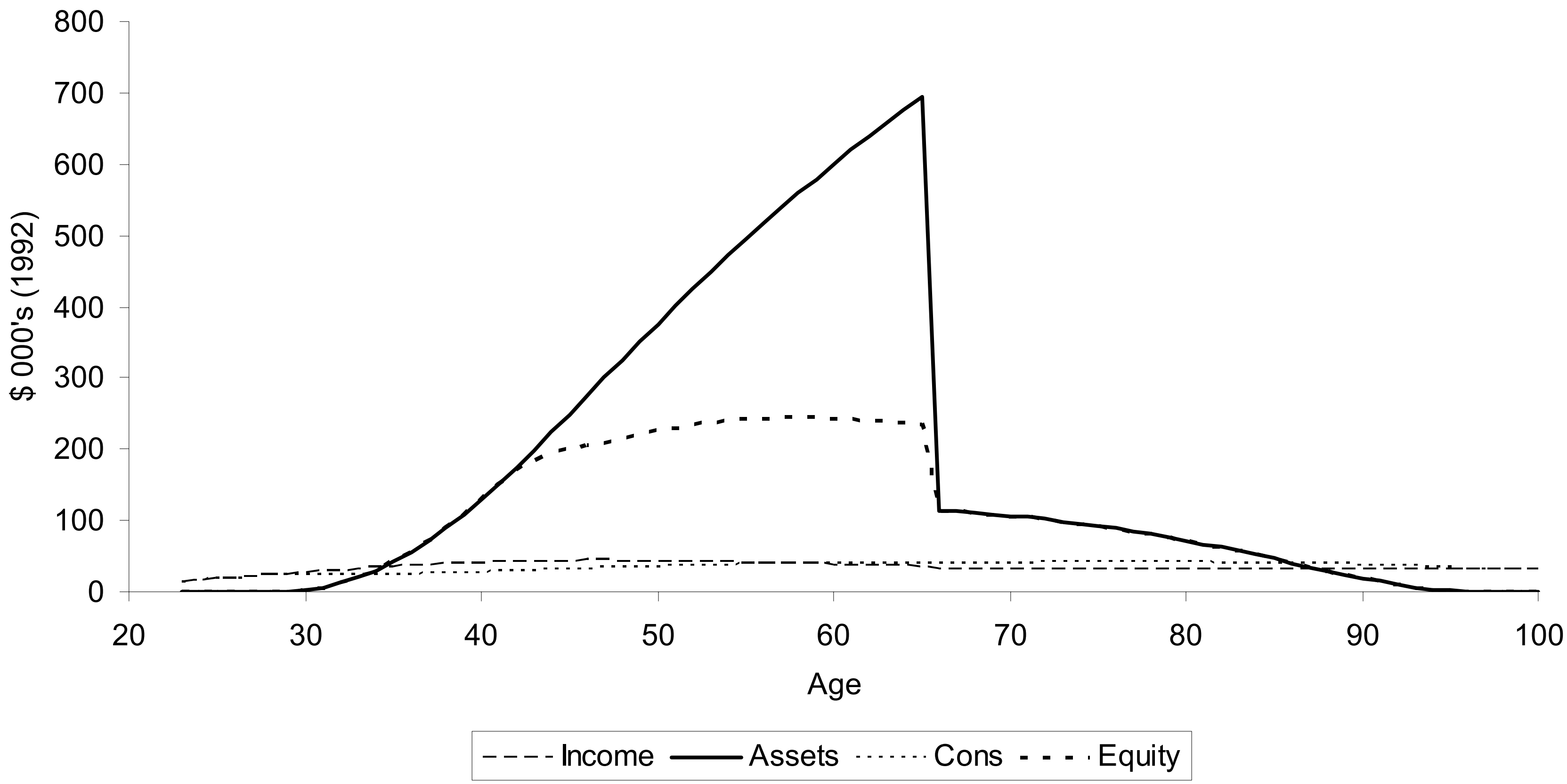
Dotted line represents wages without pension. If a pension is offered, worker productivity may improve (possibly because of the incentive effects, reduced turnover or sorting effects of the pension), implying an increased total wage, shown as the top solid line. The worker will have pension contributions (calculated at the risk free rate) deducted from wages, leaving cash wages as shown by the bottom solid line. To the worker, however, it will appear as though cash wages have fallen by the smaller amount shown by the braces – equivalent to calculating pension contributions at a higher discount rate than the risk free rate.

Figure 2: Assumed earnings profile



Source: Campbell *et al.* (1999). This graph consists of polynomial smoothed age dummies from an analysis of average earnings from the PSID.

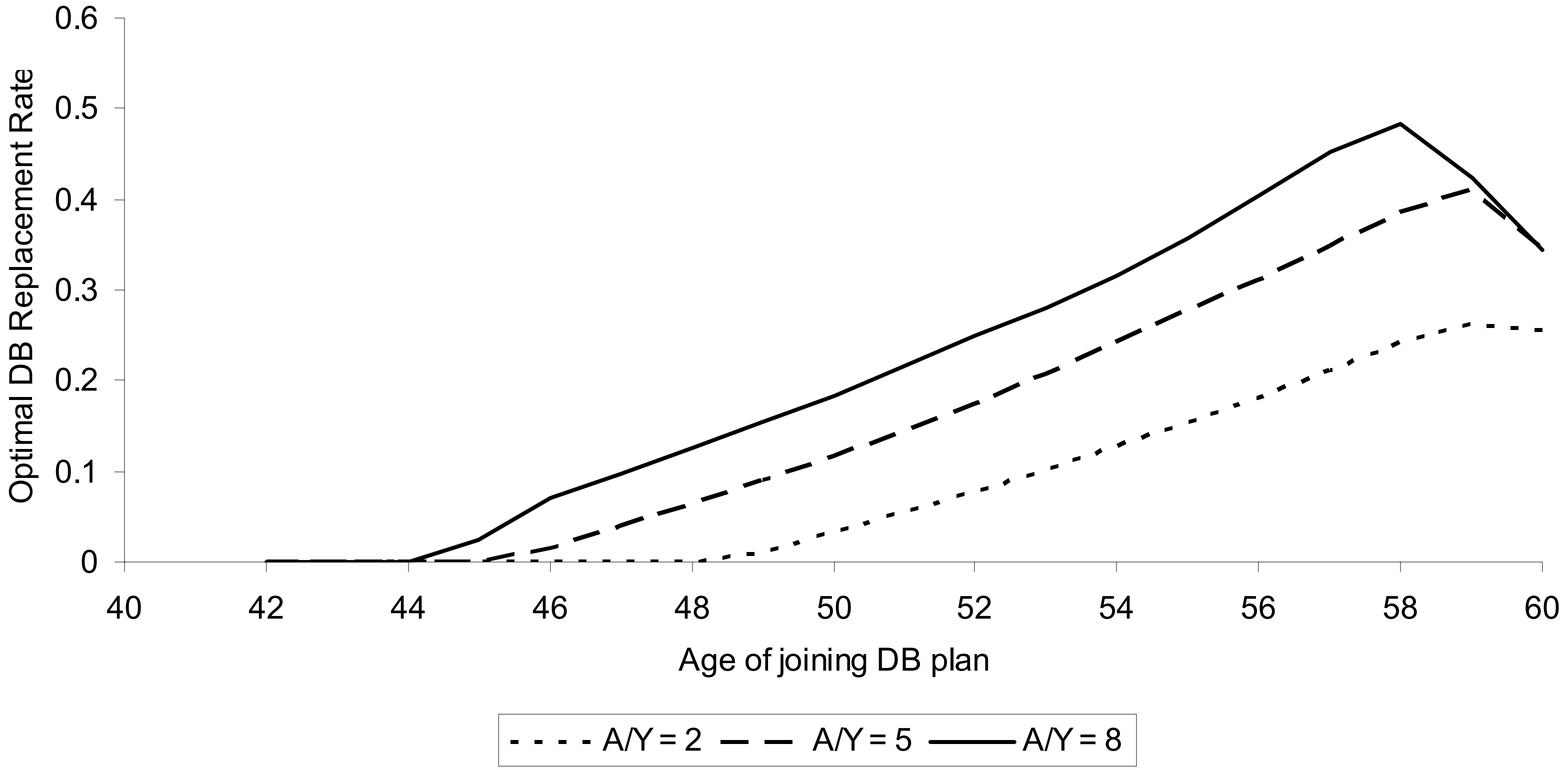
Figure 3: Optimal Lifetime Earnings Profile: No DB Plan



Notes to Figure 3:

All parameters equal expected value throughout life. Assumptions as assumed in Table 2, for college-educated individual. $\kappa = 0$. 'Assets' line includes direct equity and bond holdings. 'Equity' line includes direct equity holdings. 'Income' includes proceeds of private annuity in retirement. 'Cons' is consumption.

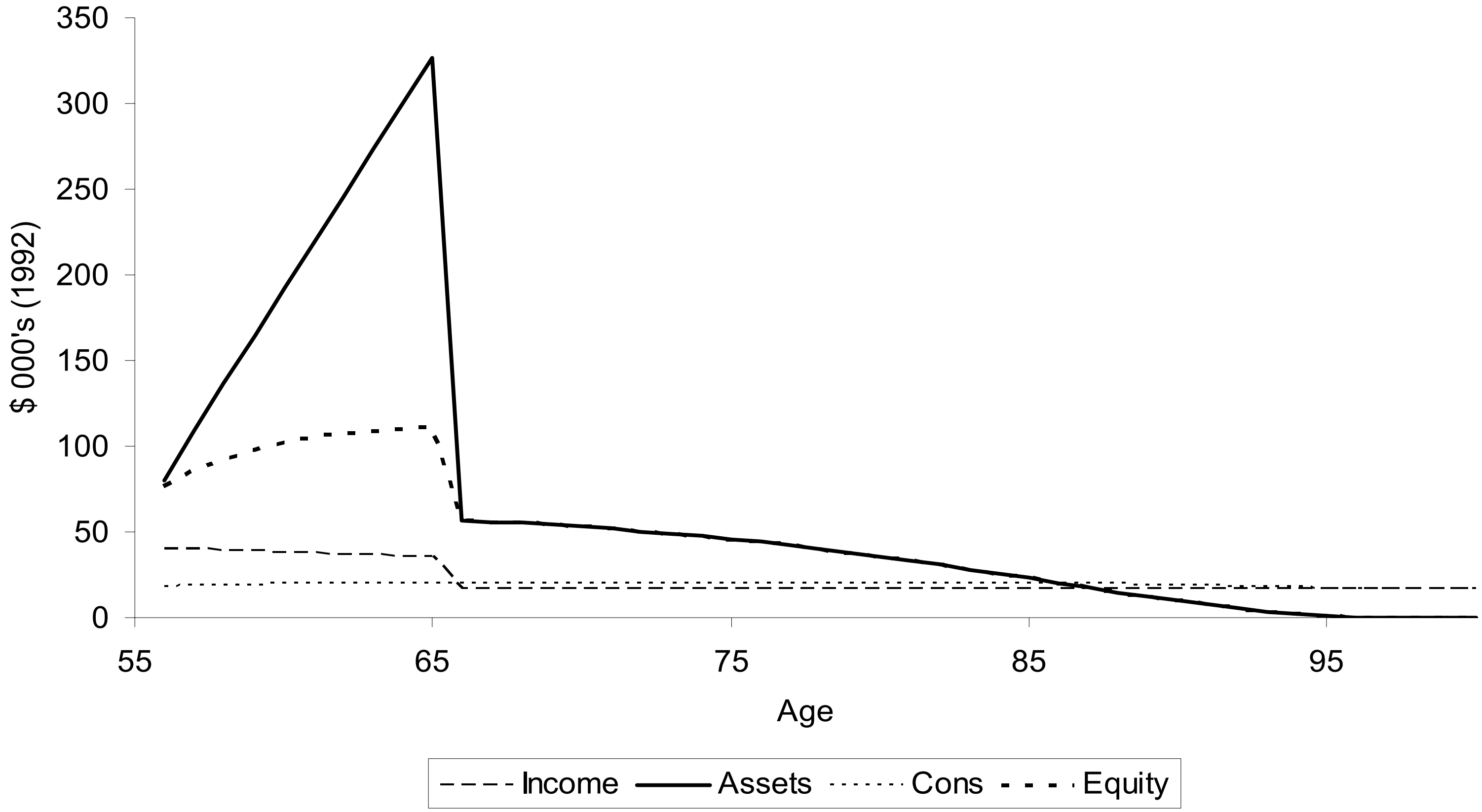
Figure 4: Optimal DB replacement rate by time to retirement and initial wealth



Notes to Figure 4:

Vertical axis shows optimal DB replacement rate. In the notation of the model, this is $t\hat{\kappa}$. Horizontal axis shows the age at which the individual is offered the option of joining the DB plan. A/Y means initial financial assets expressed as a ratio to initial wage income. At ages lower than 44, the optimal participation in the DB plan is 0 for all levels of wealth less than the maximum shown here. The values are shown in Table 3.

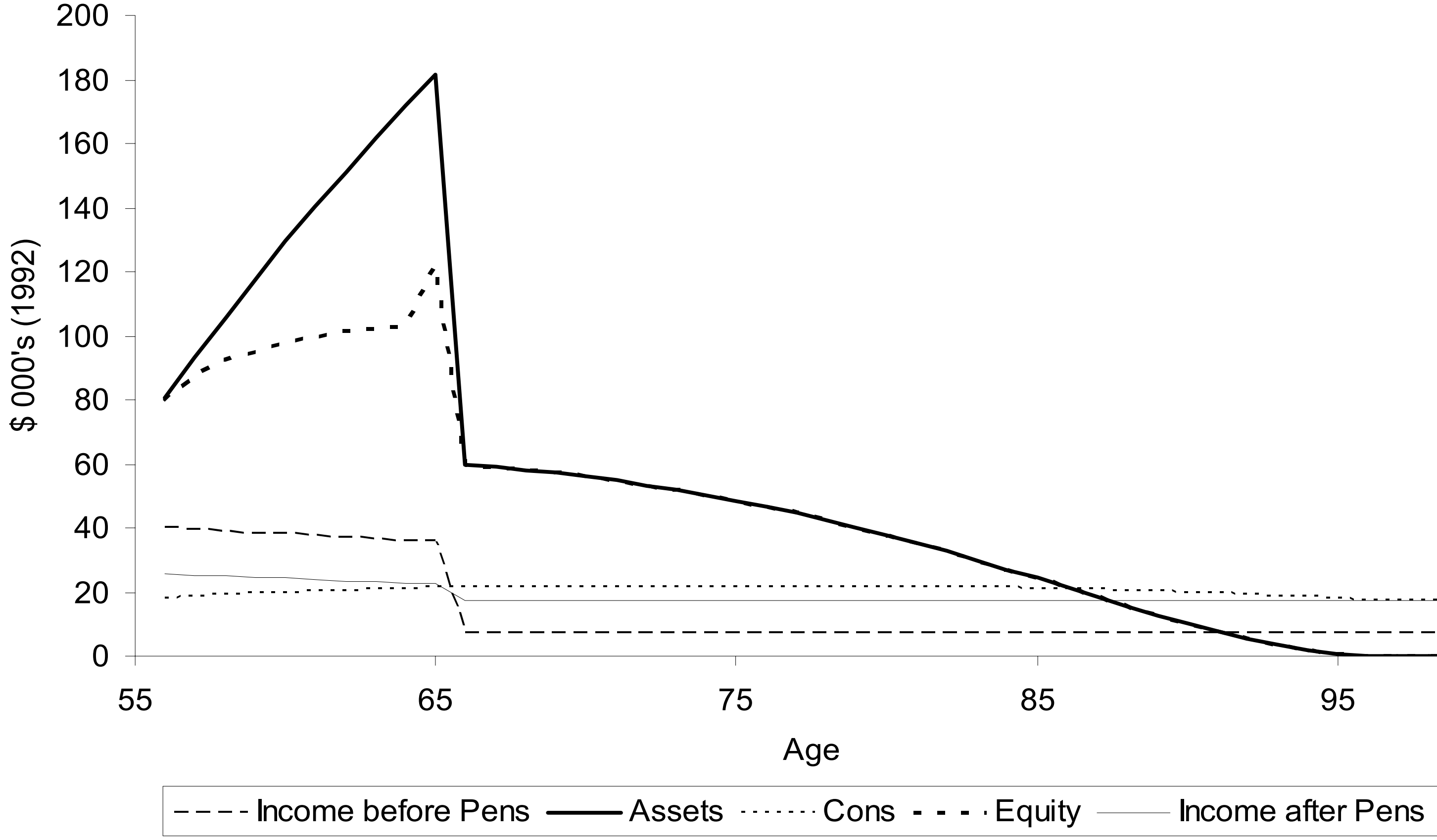
Figure 5: Age-Consumption Profiles: Base Case, $\kappa = 0$.



Notes to Figure 5:

All parameters equal expected value throughout life. Assumptions as assumed in Table 2, for college-educated individual. $\kappa = 0$. 'Assets' line includes direct equity and bond holdings. 'Equity' line includes direct equity holdings. 'Income' includes proceeds of private annuity in retirement. 'Cons' is consumption.

Figure 6: Age-Consumption Profiles: Base Case, $\kappa = \hat{\kappa}$.



Notes to Figure 6:

All parameters equal expected value throughout life. Assumptions as assumed in Table 2, for college-educated individual. $\kappa = \hat{\kappa}$. 'Assets' line includes direct equity and bond holdings. 'Equity' line includes direct equity holdings. 'Income after Pens' line shows income after pension contributions and pension annuity is paid out. 'Income before pens' includes proceeds of private annuity in retirement, but not the pension annuity. 'Cons' is consumption.