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Secular Changes in Wealth Inequality and Inheritance

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Abstract

Data suggest the distribution of wealth among households in the United States and the United Kingdom has become more equal over the last century — though the pattern may have reversed recently. This paper shows that a model in which all households save for life–cycle reasons and some for dynastic purposes as well offers a possible explanation: the model predicts rising cross–sectional equality of wealth when longevity increases. In terms of recent changes, the model suggests that expansion of social security programs and government debt can lead toward more wealth inequality, and that slower growth may do the same.

Keywords: wealth distribution; social security; inheritance; altruism; life cycle saving; longevity

JEL classification numbers: D31; E21: H55; H6; D1

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Secular Changes in Wealth Inequality and Inheritance

Life–cycle and bequest–related saving seem to occur together in practice, and this paper attempts to demonstrate that an economic model incorporating both has advantages over conventional analytical approaches which specialize to only one.[1] In particular, with a compound model one can study the consequences of exogenous shifts in the relative importance of the two motives for saving, and that constitutes the present paper's focus. This paper shows how such shifts can affect cross–sectional wealth inequality, and it suggests possible interpretations for several empirical puzzles.

Darby (1979, c.3) presents one puzzle. He notes that life and retirement spans in the U.S. have lengthened considerably in the last century, that one would think this should have substantially increased life–cycle saving, but that data shows a roughly constant aggregative saving rate. Darby concludes that life–cycle saving may not be quantitatively important. In contrast, Section 3 below shows that the present paper's hybrid model can simultaneously allow a substantial role for life–cycle saving and an increase in that role without a corresponding change in aggregative wealth accumulation.

Second, data show the U.S. distribution of wealth is more unequal than the distribution of earnings (Diaz–Gimenez *et al.*, 1997). Although the life–cycle model is broadly consistent with such a relation, it does not seem able to predict the very large wealth shares of, say, the richest 1 and 5% of U.S. households (Huggett, 1996). Section 6 below shows that the present paper's model, on the other hand, can explain a very concentrated upper tail for the wealth distribution.

Third, various sources suggest that a number of countries have experienced changes in their wealth distributions during the twentieth century. Wolff (1996), for example, finds a reduction in wealth inequality in the last 75 years for the U.S., U.K., and Sweden — perhaps followed by an upturn after 1980 in the United States. Atkinson *et al.* (1989) find even sharper reductions for British data 1923–81. Surprisingly, our model shows how this secular pattern may be related to Darby's observations on demographic change.

In this paper's model, some family lines, specifically, a fraction λ, are altruistic in the sense of caring about the utility of their adult children and subsequent descendants. Such households may choose to accumulate estates for bequests. Nonaltruistic families care solely about their own lives. A mixture of bequest–motivated and life–cycle saving can therefore emerge. The model shows that bequest–motivated wealth accumulation will tend to be highly interest elastic and, when there is a mixture of saving behaviors, will tend to cause wealth inequality as altruists save more than nonaltruists. Analysis then implies that demographic changes which increase incentives for life–cycle saving need not affect the economy's interest rate, which interest sensitive dynastic behavior sets. However, as life–cycle accumulations rise despite the economy's overall capital stock remaining the same, the composition of overall saving must adjust, with estate building declining in relative significance. The latter shift can, in time, diminish wealth inequality.

[1] The importance of life–cycle saving seems evident (Modigliani, 1988). For discussions of the possible quantitative significance of intergenerational transfers, see Kotlikoff and Summers (1981), Kotlikoff (1988), and Gale and Scholz (1994).

The idea that only some family lines manifest altruism is necessary for this paper's principal outcomes. Empirically, many households do not seem connected to their descendants through positive intergenerational transfers (Altonji *et al.*, 1992, 1997; Laitner and Ohlsson, 2001; Laitner and Juster, 1996). A simple explanation would be that preference orderings differ among families. That is this paper's viewpoint: this paper assumes that λ is exogenously given and is neither 0 nor 1.[2] Alternatively, all families may be altruistic but earning ability differences may induce only some to want to leave positive bequests. In other words, high earners may desire to share with their descendants through intergenerational transfers, whereas low earners may expect their descendants to have higher consumption than they do without their assistance. Laitner (1992), Fuster (1998), and Nishiyama (2000) examine such frameworks. The present paper's model has the virtues of being simpler and more convenient to analyze, and it may help to develop predictions and intuitive explanations of comparative static results for the complicated, stochastic systems.

In this paper's model all households save for life–cycle reasons but, as stated, some have dynastic time horizons as well. Sections 1–5 present a theoretical analysis of the basic framework and examine the possible effects of changes in mortality, social security, and the rate of technological progress. They demonstrate that the model is tractable for studying both long and short–run issues. Section 6 presents a calibrated numerical example, and it simulates the possible quantitative impacts of recent policy and demographic changes.

1. A Simple Model

For expositional simplicity, Sections 1–5 assume two–period lives. Throughout, this paper assumes that within each birth cohort a set fraction $1-\lambda$ of households save for life–cycle purposes alone, and that the remaining fraction λ desire both life–cycle saving and an estate. Children of bequeathors become bequeathors themselves. Section 6 elaborates the model to include realistic life spans.

Household Behavior. Suppose each household lives two periods, inelastically supplying one unit of labor in its first period of life, and spending its second period in retirement. A household raises children during its first period. The next period the children form their own households and pass their first period of adult life — while their parents are retired. For simplicity, think of one–adult households, each raising one child.[3] There is exogenous labor–augmenting technological progress at rate $g-1$: a household born at t supplies g^t "effective labor units" in its youth. The wage per effective labor unit is W_t. Until Section 6, assume all households of the same birth cohort have the same earning ability.

Conditional on receiving inheritance i_t and leaving bequest i_{t+1}, a household born at t has lifetime utility

$$u(i_t, i_{t+1}, t, W_t, R_{t+1}) = \max_{c_{1t}, c_{2t} \geq 0} \{(1-\theta)\cdot v(c_{1t}) + \theta \cdot v(c_{2t})\}, \tag{1}$$

[2] Woodford (1986) and Michel and Pestieau (1998), for example, also use heterogeneous preferences.

[3] Laitner (1991) shows that this type of formulation is equivalent to having 2–adult households, 2 children per household, and (strictly) assortative mating.

$$\text{subject to:} \quad c_{1t} + \frac{c_{2t}}{R_{t+1}} \leq W_t \cdot g^t + i_t - \frac{i_{t+1}}{R_{t+1}}, \tag{2}$$

where c_{1t} is the household's first period of adult life consumption, c_{2t} is its second–period consumption, the price of the single consumption good is always 1, R_{t+1} is one plus the interest rate on savings carried from period t to $t+1$, and $\theta \in (0,1)$ registers the weight households put on consumption in old age relative to youth. All households have the same θ. If households are impatient, θ will be small; similarly, if, for example, minor children receive a large allocation of households' total resources, $1-\theta$ will tend to be high. To generate the full class of concave, additively separable, homothetic utility functions, one sets

$$v(c) = \frac{c^\gamma}{\gamma}, \quad \gamma < 1 \quad \text{or} \quad v(c) = \ln(c). \tag{3}$$

For algebraic simplicity, the first five sections of this paper restrict themselves to the logarithmic case, corresponding to $\gamma = 0$. Most results carry over simply for $\gamma \neq 0$, as the comments below indicate. Performing the maximization in (1)–(2) for the logarithmic case,

$$c_{1t} = (1-\theta) \cdot [W_t \cdot g^t + i_t - \frac{i_{t+1}}{R_{t+1}}] \quad \text{and} \quad c_{2t} = \frac{\theta}{1-\theta} \cdot R_{t+1} \cdot c_{1t}. \tag{4}$$

For the fraction of households who are not altruistic, set $i_t = i_{t+1} = 0$. Expression (4) characterizes their behavior. Letting s_{1t} be the saving of young, nonaltruistic households,

$$s_{1t} = \theta \cdot W_t \cdot g^t. \tag{5}$$

Households which are altruistic have the same lifetime utility function and, conditional on their inheritances and bequests, solve the same lifetime problem. This paper assumes that institutions preclude negative intergenerational transfers and that a dynasty chooses inheritances to solve

$$\max_{i_{t+1} \geq 0} \sum_{t=0}^{\infty} \xi^t \cdot u(i_t, i_{t+1}, t, W_t, R_{t+1}), \tag{6}$$

where (1)–(2) determine $u(.)$. The intergenerational subjective discount factor is $\xi < 1$. In terms of timing, an altruistic household born at t receives inheritance i_t as it begins its first period of life; it then chooses its first period of life consumption, say, c_{1t}^d, and saving, say, s_{1t}^d; as its second period of life begins, its wealth plus interest is $R_{t+1} \cdot s_{1t}^d$, and it divides this sum between a bequest i_{t+1} to its grown child and its own retirement consumption c_{2t}^d.

A dynasty's first–order condition for i_{t+1} is

$$u_2(i_t, i_{t+1}, W_t, R_{t+1}) + \xi \cdot u_1(i_{t+1}, i_{t+2}, W_{t+1}, R_{t+2}) \leq 0 \quad \text{all} \quad t,$$

with equality when $i_{t+1} > 0$. Using the envelope theorem, the preceding condition becomes

$$\frac{1}{R_{t+1}} \cdot v'(c_{1t}^d) \geq \xi \cdot v'(c_{1,t+1}^d), \text{ equality for } i_{t+1} > 0.$$

Or, since utility is logarithmic,

$$c_{1,t+1}^d \geq R_{t+1} \cdot \xi \cdot c_{1t}^d, \text{ equality for } i_{t+1} > 0. \tag{7}$$

As stated, altruistic parents beget altruistic children, while children with nonaltruistic parents are themselves nonaltruistic.

For a dynastic household started at t, saving carried from youth to old age, s_{1t}^d, is the sum of two components. One is life–cycle saving, given by θ times the right–hand side of (2). The other is estate–motivated saving. Combining them,

$$s_{1t}^d = \theta \cdot [W_t \cdot g^t + i_t - \frac{i_{t+1}}{R_{t+1}}] + \frac{i_{t+1}}{R_{t+1}}. \tag{8}$$

Our analysis assumes that in the initial time period, each dynastic household receives the same inheritance. Given identical preferences and earnings, one dynasty's subsequent bequests are the same as another's.[4]

Let the total net assets which the household sector carries from time t to $t+1$ be A_{t+1}. Then letting the total number of households initiated at each date be N, accounting implies

$$A_{t+1} = N \cdot (1-\lambda) \cdot s_{1t} + N \cdot \lambda \cdot s_{1t}^d. \tag{9}$$

Production Sector. The economy has an aggregate production function

$$Q_t = [K_t]^\alpha \cdot [E_t]^{1-\alpha}, \quad \alpha \in (0,1),$$

where Q is GDP, K is the aggregate capital stock, and E is the "effective" labor supply. The latter is

$$E_t = N \cdot g^t. \tag{10}$$

GDP is homogeneously divisible into consumption and investment goods. All capital which firms use at time $t+1$ must have been built in prior periods and financed from t to $t+1$. Letting μ be the rate of physical depreciation, competitive behavior yields

$$R_t = 1 + \alpha \cdot [K_t]^{\alpha-1} \cdot [E_t]^{1-\alpha} - \mu \quad \text{and} \quad W_t = (1-\alpha) \cdot [K_t]^\alpha \cdot [E_t]^{-\alpha}. \tag{11}$$

[4] In the steady–state analysis below, differences among dynastic households are not interesting since the initial distribution of inheritances remains unchanged forever. The distribution would change somewhat, on the other hand, during transitions between steady states (Caselli and Ventura, 1996). See also Section 6.

General Equilibrium. Assume that the economy is closed and, at this point, that there is no government sector. Then household net worth must exactly finance the physical capital stock. In other words,

$$A_t = K_t \text{ all } t. \tag{12}$$

2. Steady–State Growth

Define a steady state equilibrium (an SSE) to be an equilibrium for the economy with (i) a constant interest factor R and (ii) geometric growth at constant rates for Q, K, E, and W. Condition (11) immediately shows that K_t/E_t and W_t must be constant if R_t is, and (10) shows that E has growth factor g; thus, K_t and Q_t have steady–state growth factor g. It remains to find the constant value(s) of R at which saving and investment — or the stock of wealth and the stock of capital — are equal. We study the last condition using a picture (as in Tobin (1967)).

Fig. 1 considers the steady states of a purely life–cycle economy (i.e., $\lambda = 0$). With only life–cycle saving, (5) and (9)–(10) yield

$$\frac{A_{t+1}}{W \cdot E_t} = \frac{N \cdot s_{1t}}{W \cdot E_t} = \theta. \tag{13}$$

The latter determines the "household wealth supply curve," Fig. 1's H–curve.

The downward sloping "production sector curve," the P–curve in Fig. 1, comes from the aggregate production function and definition of an SSE. In a steady state, $W_t = W$, $R_t = R$, and $K_{t+1} = g \cdot K_t$ all t; combining these with (11), for any SSE

$$\begin{aligned} \frac{K_{t+1} \cdot (R + \mu - 1)}{W \cdot E_t} = g \cdot \frac{K_t \cdot (R + \mu - 1)}{W \cdot E_t} = g \cdot \frac{\alpha}{1 - \alpha} \iff \\ \frac{K_{t+1}}{W \cdot E_t} = g \cdot \frac{\alpha}{1 - \alpha} \cdot \frac{1}{R + \mu - 1}. \end{aligned} \tag{14}$$

General equilibrium requires $K_{t+1} = A_{t+1}$; so, for an SSE we must be at the intersection e of H and P in Fig. 1. Fig. 1 shows there is a unique SSE when $\lambda = 0$.

Fig. 1: The SSE demand (P) and supply (H) of wealth in the pure life–cycle case

Turning to the more general model with some dynastic households (i.e., $\lambda > 0$), we first examine the time trend of inheritances. In a steady state, K and Q grow with factor g. To maintain the capital stock, time–t gross investment must then equal $(g-1+\mu)\cdot K_t$, which grows with factor g. Since national output equals consumption plus investment, consumption must also have steady–state growth factor g. Line (4) shows that life–cycle consumption has this growth factor as well; hence, using the second half of (4) first, c^d_{1t} and c^d_{2t} do too. Looking at (7), we can then see that in any SSE,

$$g \geq R \cdot \xi.$$

There are two cases. Either

$$g > R \cdot \xi, \tag{15}$$

in which event the first order condition for i requires $i_{t+1} = 0$ all t; or,

$$g = R \cdot \xi, \tag{16}$$

in which case i_{t+1} can be positive. As stated, dynastic consumption has growth factor g in an SSE. The present value of a dynasty's consumption from date $t+1$ forward equals $R \cdot s^d_{1t}$ plus the present value of current and future earnings. Earnings grow with factor g; hence, s^d_{1t} grows with factor g. The accounting relation

$$c^d_{2t} + i_{t+1} = R \cdot s^d_{1t}$$

then shows that in an SSE,

$$i_t = i_0 \cdot g^t \quad \text{all} \quad t. \tag{17}$$

Collecting these characterizations, and noting that our dynastic results hold trivially in a steady state with zero inheritances (i.e., in a steady state with (15) instead of (16)),

Proposition 1: *In any SSE, c^d_{1t}, c^d_{2t}, c_{1t}, c_{2t}, s_{1t}, s^d_{1t}, and i_t must grow at rate $g-1$. In a SSE with positive inheritances, equality (16) must hold. In a SSE without positive inheritances, provided there are at least some dynastic households, inequality (15) must hold.*

In other words, in a steady state, dynasties behave exactly in accordance with the permanent income hypothesis: during its lifetime, each dynastic household consumes its earnings plus $R \cdot i_t - g \cdot i_t$, the interest on its inheritance less what is needed to maintain the magnitude of the latter's principal relative to future earnings. This description applies regardless of the magnitude of i_0.

Fig. 2 graphs the steady–state wealth supply curve H for an economy with $\lambda > 0$. For a steady state, (8)–(9) and (17) yield

$$\frac{A_{t+1}}{W \cdot E_t} = (1-\lambda)\cdot\theta + \lambda \cdot [\theta \cdot (1 + \frac{i_0}{W}) + (1-\theta)\cdot\frac{g}{R}\cdot\frac{i_0}{W}] \quad \text{every} \quad t. \tag{18}$$

If $i_0 = 0$, which must be the case for $R < g/\xi$, there are only life–cycle wealth accumulations, and the curve resembles Fig. 1, with the right–hand side of (18) equaling θ. If

$R = g/\xi$, different values of i_0 yield the horizontal segment in the H curve — as (18) shows, a larger i_0 moves us further to the right; as the preceding paragraph notes, any i_0 is compatible with equilibrium behavior.

Continuing to refer to Fig. 2, when the production–sector curve is $\bar{P}$, crossing H below $R = g/\xi$, there exists a unique SSE at $\bar{e}$ with no inheritances or bequests. If the demand curve is P, there is a unique SSE at e. In the latter case, the horizontal coordinate at f gives the life–cycle saving contribution to total wealth accumulation, and the horizontal difference between e and f measures the contribution of bequest–motivated saving. These are the only two possible outcomes.

Fig. 2: The SSE demand and supply of wealth with dynastic saving

Summarizing,

Proposition 2: *Assume logarithmic utility and $\lambda > 0$. Then if an SSE with positive inheritances exists, it is unique and the steady–state interest rate is $R = g/\xi$. If no such SSE exists, there is a unique steady state with only life–cycle saving, $R < g/\xi$.*

In the more general isoelastic case from (3), the vertical segment of H would instead have a slope — positive if $\gamma > 0$, and negative if $\gamma < 0$. The section to the right of f would still be flat, but its height would be the root R of

$$(R \cdot \xi)^{1/(1-\gamma)} = g.$$

The analysis would otherwise be the same.

Suppose we measure the degree of inequality in the distribution of wealth with a Gini coefficient. Think about the end of period t, households having had time to complete their current labor and consumption, and elderly households to make bequests. Then all N elderly households have 0 net worth; the $(1 - \lambda) \cdot N$ middle–of–life nondynastic households each have net worth s_{1t}; and the $\lambda \cdot N$ middle–of–life dynastic households each have $s^d_{1t} \geq s_{1t}$. Fig. 3 shows the Lorenz curve, $ABFE$. From A to B, a distance of $1/2$, we have the elderly; from B to C, a distance of $(1 - \lambda)/2$, the middle–of–life nondynastic households; and, from C to D, a distance of $\lambda/2$, the middle–of–life dynastic households. With either $\lambda = 0$ or $\lambda = 1$, the Gini coefficient would equal twice the area of triangle ABE; thus, the Gini would be $1/2$. For other values of λ, we must add twice the shaded area. The shaded area equals

$$\frac{1}{2} \cdot FG \cdot [BC + CD] = \frac{1}{2} \cdot FG \cdot BD = \frac{1}{2} \cdot FG \cdot \frac{1}{2}.$$

Since the slope of $\overline{BE}$ is 2, $CG = 1 - \lambda$. The height of F gives the fraction of total wealth held by middle–of–life nondynastic households; hence,

$$FG = (1 - \lambda) - \frac{(1 - \lambda) \cdot s_{1t}}{(1 - \lambda) \cdot s_{1t} + \lambda \cdot s_{1t}^d} \ .$$

Then the Gini, including the shaded area, is

$$\frac{1}{2} + \frac{1}{2} \cdot (1 - \lambda) \cdot \frac{(1 - \lambda) \cdot s_{1t} + \lambda \cdot s_{1t}^d - s_{1t}}{(1 - \lambda) \cdot s_{1t} + \lambda \cdot s_{1t}^d} \ . \tag{19}$$

Referring back to Fig. 2, let e_x be the x–coordinate of equilibrium point e, and let f_x be the x–coordinate of point f. Equilibrium condition (12) shows

$$e_x = \frac{[(1 - \lambda) \cdot s_{1t} + \lambda \cdot s_{1t}^d] \cdot N}{W \cdot g^t \cdot N}.$$

From (5),

$$f_x = \frac{N \cdot s_{1t}}{W \cdot g^t \cdot N}.$$

So, (19) yields

$$\text{Gini} = \frac{1}{2} + \frac{1}{2} \cdot (1 - \lambda) \cdot \frac{e_x - f_x}{e_x}. \tag{20}$$

Notice that the (steady–state) Gini is not dependent on time.

Fig. 3: The Lorenz curve ($ABFE$) for the cross–sectional distribution of wealth

The intuition for (20) is clear. Dynastic households have higher net worth than purely life–cycle households because the former save to maintain their family line's wealth as well as for their own retirement, while the latter save just for retirement. The distance between f and e in Fig. 2 shows the margin by which aggregate wealth exceeds accumulation in

a purely life–cycle economy. For a given λ, the more dynastic wealth which equilibrium requires, the wider the margin between e and f, and the more overall wealth inequality. For a given percentage margin of e_x over f_x, a lower λ implies a smaller subset of households provides the requisite extra wealth, so that each of the latter households must be richer, and the degree of inequality correspondingly greater.

Summarizing,

Proposition 3: *Suppose our model has a SSE with positive bequests, as at point e in Fig. 2. Then (20) gives the Gini coefficient for wealth inequality at the close of each period. In a SSE, the Gini is time–invariant.*

3. Longer Lives

Darby's (1979) tables 1–2 show that the probability of surviving to 65 has increased substantially in the last century, as have expected remaining life spans for individuals surviving to 65. In terms of our life–cycle model, these factors presumably raise θ, which determines the fraction of lifetime consumption that households allocate to retirement. This section examines the implications of such a change.

A larger θ increases the life–cycle wealth accumulation of young households. In terms of Fig. 2, the vertical section of the H curve shifts to the right. On the other hand, θ plays no role in (16), which determines the height of the H–curve's flat section. So, the flat section's height is unchanged. Fig. 4 illustrates. If the old SSE was at $\bar{e}$, with no inheritances, the new one, shown as $\bar{\bar{e}}$, has a lower interest rate and higher capital–to–labor ratio. If, however, the old SSE was e, and if the increase in θ is not too big, the new steady state lies again at e, with the same interest rate and capital intensivity. The latter is the case of interest in this paper.

Fig. 4: The H–curve shifts because of an increase in longevity

At e, when θ rises, the distribution of wealth becomes more equal. Fix $\lambda > 0$. As life–cycle saving increases with a rise in θ, the vertical part of H, and with it point f, shifts to the right. Proposition 3 shows that as this happens, the Gini coefficient for wealth falls: dynastic wealth holdings adjust to restore (16); when demographic changes raise life–cycle accumulations, the equilibrium extra net worth that dynastic households carry for bequests falls — and the latter is the important source of the economy's wealth inequality.

Summarizing,

Proposition 4: *Suppose our model has a SSE with positive bequests. Then an increase in θ sufficiently small to allow a new SSE with positive bequests leaves the steady–state interest rate and the capital–to–effective labor ratio unchanged. However, the distribution of wealth becomes more equal.*

Darby attacks the life–cycle model on the basis of demographic changes: if saving is explained by the life–cycle model, the economy's capital intensity, he argues, should have risen substantially over the course of the last century as longevity increased. He is thinking of the pure life–cycle equilibrium, $\bar{e}$, in Fig. 4. At e, on the other hand, with both life–cycle and bequest–motivated saving, increasing longevity raises the relative importance of life–cycle saving, though not, in the long run, the size of the overall capital stock or the steady–state interest rate.

Going further, Wolff and Atkinson find a secular decline in empirical cross–sectional wealth inequality, and Proposition 4 suggests an unexpected connection to Darby's analysis: if increases in longevity have made life–cycle saving progressively more important, they may simultaneously have reduced the disequalizing role of inheritances. Section 6 shows the possible quantitative importance of this point.

Although we could examine the adjustment path from the old SSE to the new one after a change in θ, we postpone our discussion of dynamic analysis until Section 4.

4. Government Debt and Social Security

Wolff's U.S. data not only shows a secular decline in wealth inequality but also a recent upturn, and this section suggests one possible explanation for the latter: government policies may inadvertently have increased the relative weight of dynastic savings.[5] Since the U.S. social security system's inception in the 1930s, benefits (and taxes) have risen as periods of retirement became longer and more prevalent, as the system expanded to include a larger fraction of the workforce, and as Congress raised statutory benefits. It is also the case that the U.S. national debt rose very rapidly in the early 1940s, and, after a period of gradual decline, rapidly again in the 1980s. Our model predicts that growth either in public debt or in the size of an unfunded social security system will, cet. par., tend to increase wealth inequality among households.

Consider first an unfunded social security system which taxes labor earnings in order to pay benefits to retirees. Let the tax be a proportion τ_{ss} of earnings. The right–hand side of household budget (2) becomes

$$W_t \cdot g^t \cdot (1 - \tau_{ss}) + \tau_{ss} \cdot \frac{W_{t+1} \cdot g^{t+1}}{R_{t+1}} + i_t - \frac{i_{t+1}}{R_{t+1}}.$$

The first term has been modified to reflect social security taxes; the second term, which is new, registers social security benefits. Assuming a steady state, and solving for the saving of a young, purely life–cycle household,

[5] The equations in this section and results on aggregative capital accumulation closely resemble Michel and Pestieau (1998) — though the discussion of distributions does not.

$$\frac{s_{1t}}{W \cdot g^t} = \theta \cdot [1 - \tau_{ss} + \frac{\tau_{ss} \cdot g}{R}] - \frac{\tau_{ss} \cdot g}{R}. \tag{21}$$

The first right–hand side term reflects private provision for retirement consumption; the second term reflects the fact that households now receive external resources during retirement. Line (16) shows that $g/R = \xi < 1$ for an SSE with positive inheritances; therefore, the first right–hand side term in (21) is smaller than θ. The reduction is due to an income effect: at equilibrium interest factor R, the present value of a household's social security benefits fall short of its taxes, leading to less consumption in both periods of life, and hence less youthful saving. The second right–hand side term of (21) comes from the intertemporal transfers from earning years to retirement inherent in social security. These, of course, displace life–cycle saving.

The steady–state saving of dynastic households in youth is analogously affected, with (8) changing to

$$\frac{s^d_{1t}}{W \cdot g^t} = \theta \cdot [1 - \tau_{ss} + \frac{\tau_{ss} \cdot g}{R} + \frac{i_0}{W} - \frac{i_0}{W} \cdot \frac{g}{R}] - \frac{\tau_{ss} \cdot g}{R} + \frac{i_0}{W} \cdot \frac{g}{R}\ . \tag{22}$$

Since condition (16) is unchanged, the flat part of the H curve in Fig. 2 remains at the same height. Lines (21)–(22) show the vertical section shifts left (and it now assumes a positive slope due to the presence of R on the right–hand side of (21)). The demand curve remains unchanged. Thus, Proposition 3 shows that with positive inheritances, wealth inequality should increase when τ_{ss} does.[6]

Increases in the national debt can have the same effect. Suppose one–period bonds fund a government debt of size $B_t = B_0 \cdot g^t$ at time t, with B_t being the stock of bonds expiring at t, and that society levies lump–sum taxes of $\tau_t = \tau_0 \cdot g^t$ on young households to fund the debt's interest liability. The net worth which a nonaltruistic household carries into retirement is

$$s_{1t} = \theta \cdot [W_t \cdot g^t - \tau_t];$$

the net worth for altruistic households of the same age is

$$s^d_{1t} = \theta \cdot [W_t \cdot g^t + i_t - \tau_t] + \frac{(1-\theta) \cdot i_{t+1}}{R_{t+1}}.$$

Governmental accounting requires

$$B_{t+1} - B_t + N \cdot \tau_t = (R_t - 1) \cdot B_t\,. \tag{23}$$

Equilibrium requires

[6] This paper's simplified model overlooks, of course, possible labor supply effects from social security taxes. It also follows most existing work in focusing on the equality of the distribution of private wealth (i.e., wealth excluding capitalized social security benefits). Line (21) implies that the direction of our outcome would not change if we incorporated capitalized social security benefits into life–cycle wealth.

$$N \cdot (1-\lambda) \cdot s_t + N \cdot \lambda \cdot s_t^d = B_{t+1} + K_{t+1}. \tag{24}$$

Fig. 5 provides a picture. We change from a SSE with no government debt — having supply curve H^{old} — to a new SSE with $B_0 > 0$. As $g/\xi > g$, along the horizontal section of the H curve government debt requires positive taxes (see (23)). The latter reduce life–cycle saving, sliding point f to the left. The P curve shifts right as we add $B_{t+1}/W_t \cdot E_t$ to $K_{t+1}/W_t \cdot E_t$. The new SSE, assuming positive bequests, is e. Again, Proposition 3 shows that wealth inequality rises as B_0 does, because e and f spread further apart.

Fig. 5: The effect of a national debt on the economy's SSE

Summarizing,

Proposition 5: *Consider a steady state for our model with positive bequests. Then a small increase in social security benefits or in national debt (with taxes for debt service falling exclusively on young households) will leave the steady–state interest rate unchanged but will increase wealth inequality.*

For all of the changes which we consider, our model allows short–run analysis as well as a comparison of steady states. Fig. 6 illustrates a transition path following an increase in national debt. The experiment is as follows: at time 0 upon retiring bonds B_0, the government issues 1% more B_1 than the trend increase to $g \cdot B_0$, announcing that future debt will be $B_t = B_1 \cdot g^{t-1}$, and using the extra time–0 sales revenues for a one–period tax reduction. Prior to time 0, the economy rested in a SSE. Fig. 6 graphs adjustment paths to the new steady state for physical capital K_t, the interest rate, the dynastic inheritance i_t, and the wealth Gini, presenting percent deviations from original steady–state values. Parameters are $\alpha = .33$, $\xi = .75$, $\theta = .50$, $\lambda = .05$, $\mu = .20$, $g = 1.08$, $N = 1$, and $B_0 = .10$. The initial SSE resembles e in Fig. 4–5. Steady–state levels are $K_0 = .37$, $s_{10} = .22$, $s_{10}^d = 5.95$, and $i_0 = 6.54$. Despite the fact that they compose only about 5% of the population of family lines, dynastic savers hold roughly 63% of the economy's stock of capital and bonds. Middle–of–life dynastic households have over 26 times as much net worth as nondynastic households of the same age. The initial Gini coefficient for the cross–sectional distribution of wealth is .77.

We can determine dynamic behavior from accounting and first–order conditions; this paper's appendix provides details. The adjustment path is interesting. At time 0, the tax

reduction for young households increases life–cycle and, to a lesser extent, dynastic saving. The wealth Gini falls, and the time–1 capital stock rises. At time 1 the tax reduction is over; taxes, in fact, are above their original level because the national debt is larger. Life–cycle saving falls. Dynastic saving is temporarily low as well because the high capital stock lowers the return on assets. The capital stock falls sharply. Resulting higher interest rates raise subsequent dynastic saving and inheritances. As in Fig. 5, dynastic wealth accumulation eventually restores the capital stock to its original level — despite the larger national debt and permanently lower life–cycle saving. The final Gini is about .08% higher than prior to the policy change.

In addition to showing that our model can be saddlepoint stable with rational expectations, the dynamic example illustrates a difference between our model and Barro's (1974) well–known analysis. Both predict that changes in national debt or social security will leave the long–run interest rate and capital intensivity of the economy unaffected. However, in Barro's framework such policy changes make no difference in the short run either. Following an increase in the national debt, for instance, households perceive greater future tax liabilities, and all raise their savings accordingly, providing just enough incremental financing to preserve the old interest rate. In contrast, in this paper's model only a fraction of households are dynastic. Although dynastic families respond analogously to Barro's, purely life–cycle households do not. The impact on saving is, in effect, only λ times as much as in Barro's model. In the short run, physical capital is crowded out and interest rates rise. The latter induces dynastic households to go further, accumulating additional wealth until the old equilibrium interest factor reemerges. The extra efforts of dynastic households permanently change our model's distribution of wealth.

5. Slower Growth

The rate of technological progress in the United States and Western Europe seemed to slow down after 1970, and we might ask how according to our model that will, eventually at least, affect the distribution of wealth. This section shows our simple model predicts that slower growth implies higher steady–state wealth inequality.

Fig. 7: The effect of slower technological change on the economy's SSE

If g falls, production relation (14) shows that the demand for capital curve shifts to the left — from P^{old} to P in Fig. 7 — leading, *cet. par.*, to lower wealth inequality. Intuitively, to sustain a steady state, the economy must provide enough saving for the capital stock to grow in step with the effective labor supply; when g is lower, this is more easily accomplished, requiring less dynastic saving. However, if g falls, (16) shows R and the corresponding flat section of our household wealth supply curve will fall as well. In other words, a benefactor requires less inducement to bequeath to descendants whose consumption opportunities are growing less rapidly. This leads to a higher capital to labor ratio, shifting the equilibrium point to the right, and tending to increase the inequality of wealth holdings.

On balance, in our simple model a lower g leads to more steady–state wealth inequality. This can be established algebraically as follows. Combining (14) and (16),

$$e_x = \frac{K_{t+1}}{W_t \cdot E_t} = g \cdot \frac{\alpha}{1-\alpha} \cdot \frac{1}{g/\xi + \mu - 1}.$$

The right–most term is decreasing in g:

$$\frac{\partial e_x}{\partial g} < 0.$$

In terms of Fig. 7, this means that following a decline in g, the new equilibrium, e', must be to the right of the old one, e. Equation (20) then shows the Gini must increase. Summarizing,

Proposition 6: *Consider a steady state for our model with positive bequests. Then a small decrease in g will cause the steady–state interest rate to fall and the degree of inequality in the cross–sectional distribution of wealth to rise.*

Proposition 6, of course, presents comparisons of steady–state equilibria. The recent changes in Wolff's data mentioned in the introduction may be too soon after the 1970 slowdown for Proposition 6 to be applicable.[7]

It is also true that the calculations for Proposition 6 appear more fragile than Propositions 4–5. On the one hand, the size of γ in utility function (3) becomes important. As stated above, the vertical section of the H curve assumes a negative slope if $\gamma < 0$ — which is often thought to be the realistic case in practice. With $\gamma < 0$, as g and R fall, life–cycle wealth accumulations climb, creating a tendency toward wealth equalization not evident in Fig. 7. Further complications arise with multiperiod life spans. One issue is that in a framework with longer lives, households of many different ages would have positive net worth, and technological progress essentially gives those born the most recently the most weight in computing aggregates. Depending on the precise nature of the distribution of wealth with respect to age, faster technological progress then could either raise or lower

[7] As in simpler growth models, a diminution of g also implies a higher SSE average propensity to save — which is certainly not evident in the U.S. national income and products account data. Possibly the slowdown is too recent for the economy to have achieved a new steady state. Or, other factors beyond the scope of this paper, such as the tremendous rise in common stock prices, may have played a role.

average life–cycle wealth. A second issue arising when there are multiperiod life spans is that slower technological change flattens each household's life–cycle earnings profile. A flatter profile causes households to begin saving for retirement earlier, tending to increase average life–cycle net worth. Again, predictions from our simple model may not be reliable.

6. Calibrated Examples

This section develops a calibrated version of our model and examines the theoretical implications of Sections 2–5 from a quantitative standpoint. Since length of life is important to numerical outcomes, the examples allow multiperiod life spans and provide a rather detailed treatment of mortality. However, all of the analysis assumes the availability of actuarially fair annuities and life insurance. And, for the sake of computational simplicity, we limit our attention to steady–state equilibria.

Model. Time is discrete. Let n be the annual population growth factor. Assume each household begins with a single adult age 20 and raises n^{20} children. The children remain under their parent's care until age 20, at which point they form their own households. There is no child mortality. The fraction of adults remaining alive at age $s \geq 20$ is q_s, and the probability that an adult dies at the close of age s is

$$p_{s+1} = (q_s - q_{s+1})/q_s.$$

The maximal age is 99.

To generate a distribution of wealth among life–cycle and among dynastic households, we assume an exogenous, stationary distribution of earning abilities within each cohort. The earnings distribution is the same for dynastic and nondynastic households. Each adult has a (known) earning ability x, constant throughout his life. For simplicity, all descendants of an adult have the same ability that he does.[8] Let the density for the distribution of x be $f(x)$. In fact, we assume

$$\ln(x) \sim N(0, \sigma_x^2). \tag{25}$$

We continue to assume that labor hours are inelastic. Letting e_s be the product of experiential human capital and labor hours, and g be one plus the rate of annual labor–augmenting technological progress, an adult of age s, ability x, and birth date t supplies $e_s \cdot x \cdot g^{t+s}$ "effective" labor units. In our steady–state equilibria, the wage per effective labor unit is constant, $W_t = W$, as is the interest rate, $r_t = r$, the income tax rate τ, and the social security tax rate τ_{ss}.

The simulation model's equations are as follows. Letting $V(a, s, 0, x)$ be remaining lifetime utility for a household born at time 0, currently age s ($20 \leq s \leq 99$), having earning ability x, and beginning period s with net worth a, we have

[8] This assumption preserves the simplicity of deterministic analysis for dynastic behavior. For alternative approaches, see, for instance, Laitner (1992), Fuster (1998), or Nishiyama (2000).

$$V(a,s,0,x) = \max_{c_s,c_{ks},\ell_s} \{u(c_s,s) + n^{20}\cdot u^k(c_{ks},s) + p_{s+1}\cdot n^{20}\cdot U(\ell_s,s) + \beta\cdot(1-p_{s+1})\cdot V(a',s+1,0,x)\} \quad (26)$$

with

$$a' \equiv R_{s-1}\cdot a + e_s\cdot x\cdot g^s\cdot W\cdot(1-\tau-\tau_{ss}) + ssb_s\cdot(1-\frac{\tau}{2}) - c_s - n^{20}\cdot c_{ks} - p_{s+1}\cdot n^{20}\cdot \ell_s, \quad (27)$$

$$a' \geq 0, \quad (28)$$

where c_s is the consumption of the household's adult and $u(c_s,s)$ the corresponding flow of utility; where c_{ks} measures the consumption of each of the household's minor children when the adult is age $s = 20,...,39$, and $u^k(c_{ks},s)$ is the corresponding addition to the parent's utility flow from each minor child; where ℓ_s is the term life insurance which the parent purchases at ages $s = 20,...,38$ to protect each minor child, and $U(\ell_s,s)$ measures the (parent) household's utility from the minor child's consumption if the adult dies at the close of age s; where β is the subjective discount factor for lifetime utility; where $ssb_s(0,x)$ is social security retirement benefits, half of which are taxed ; and, where R_s is the net–of–tax interest factor for annuities,

$$R_s \equiv \frac{1+r\cdot(1-\tau)}{q_{s+1}/q_s}.$$

We assume that bankruptcy laws prevent households from borrowing without collateral, implying inequality constraint (28). Utility is isoelastic:

$$u(c,s) = \begin{cases} \frac{c^\gamma}{\gamma}, & \text{if } s \leq 65, \\ \upsilon^{1-\gamma}\cdot\frac{c^\gamma}{\gamma}, & \text{if } s > 65, \end{cases}$$

$$u^k(c,s) = \begin{cases} \omega^{1-\gamma}\cdot\frac{c^\gamma}{\gamma}, & \text{if } 20 \leq s \leq 39, \\ 0, & \text{if } s \geq 40, \end{cases}$$

with $\gamma < 1$. We discuss the relative weights for retirement consumption, υ, and minor children, ω, below. Define

$$R \equiv 1 + r\cdot(1-\tau).$$

For $20 \leq s < 39$,

$$U(\ell_s, s) \equiv \max_{c_{ks'}} \sum_{s'=s+1}^{39} \beta^{s'-s} \cdot u^k(c_{ks'})$$

$$\text{subject to:} \quad \ell_s = \frac{c_{k,s+1}}{R} + ... + \frac{c_{k,39}}{R^{39-s}}.$$

For ages $s \geq 39$, an adult need no longer buy life insurance to protect his minor children, and $U(., s) = 0$.

The framework of (26)–(28) applies to all households. For those which are nondynastic, letting $a(s, t, x)$ be the beginning–of–period net worth of a household born at t and currently age s, we impose

$$a(20, t, x) = 0 \quad \text{and} \quad a(100, t, x) = 0. \tag{29}$$

Note that because preferences are homothetic, when R and W are constant, solution of the analogue of (26)–(29) for $t \neq 0$ yields

$$a(s, t, x) = g^t \cdot a(s, 0, x) \quad \text{and} \quad V(g^t \cdot a, s, t, x) = g^{\gamma \cdot t} \cdot V(a, s, 0, x). \tag{30}$$

Nondynastic households solve (26)–(28) and simultaneously determine inheritances from an intergenerational maximization problem. Consider a dynastic household with adult born at t. Label this household "generation 0" in the dynasty. If the present value at age 20 of the current adult's inheritance is i_0, the dynasty computes its future inheritances from

$$\max_{i_j \geq 0} \sum_{j=1}^{\infty} (\xi \cdot n^{20})^j \cdot V(i_j - n^{20} \cdot i_{j+1}/R^{20}, 20, t + 20 \cdot j, x), \tag{31}$$

with ξ the intergenerational subjective discount factor.[9] Let $a^d(s, t, x)$ be the beginning–of–period net worth of a dynastic household born at t and currently age s. At this point, we will think of a minor child as receiving the present value of his inheritance at birth (though he cannot begin his own consumption until he is an adult); thus, following the notation of (31),

$$a^d(s, t + 20 \cdot j, x) = i_j \cdot R^{s-20} \quad \text{for} \quad s = 0, ..., 20, \quad \text{and} \quad a^d(100, t + 20 \cdot j, x) = 0. \tag{32}$$

As in previous sections, the fraction of households which are dynastic is an exogenous constant λ.

There is a Cobb–Douglas aggregate production function

$$Q_t = [K_t]^\alpha \cdot [E_t]^{1-\alpha}, \quad \alpha \in (0, 1),$$

[9] There are no estate taxes in the model. The U.S. estate tax has a very high credit, so that only a tiny minority of estates are liable.

where Q_t is GDP, K_t is the aggregate stock of privately owned physical capital excluding consumer durables, and E_t is the effective labor force. K_t depreciates at rate μ. The economy is closed. The price of output is always 1. Perfect competition implies

$$W_t = (1-\alpha)\cdot\frac{Q_t}{E_t} \quad \text{and} \quad r_t = \alpha\cdot\frac{Q_t}{K_t} - \mu. \tag{33}$$

Households also own a stock of consumer durables, K_t^D. The stock yields a proportionate service flow. In turn, households demand a service flow which is a fixed proportion of their total consumption. Hence, as our analysis is limited to steady states, our calibrations assume

$$K_t^D/Q_t = \text{constant}.$$

The government issues B_t one–period bonds with price 1 at time t. We assume

$$B_t/Q_t = \text{constant}.$$

Letting SSB_t be aggregate social security benefits, we assume

$$SSB_t/Q_t = \text{constant}.$$

The social security system is unfunded, so

$$SSB_t = \tau_{ss}\cdot W_t\cdot E_t. \tag{34}$$

If G_t is government spending on public consumption goods, we assume

$$G_t/Q_t = \text{constant}.$$

The government budget constraint is

$$G_t + r_t\cdot B_t + B_t = \tau\cdot[W_t\cdot E_t + r_t\cdot K_t + r_t\cdot B_t] + B_{t+1}. \tag{35}$$

Public–good consumption does not affect marginal rates of substitution for private consumption.

Normalizing the size of the time–0 birth cohort to 1 and employing the law of large numbers,

$$E_t = \sum_{s=20}^{99} n^{(t-s)}\cdot g^t\cdot q_s\cdot e_s. \tag{36}$$

Households must finance the private capital stock, government debt, and the stock of consumer durables; so,

$$K_t + K_t^D + B_t = \sum_{s=20}^{99}(n\cdot g)^{(t-s)}\cdot q_s\cdot\int_{-\infty}^{\infty}[(1-\lambda)\cdot a(s,t-s,x) + \lambda\cdot a^d(s,t-s,x)]\cdot f(x)\,dx. \tag{37}$$

In "equilibrium" all households maximize their utility and (33)–(37) hold. A "steady–state equilibrium (SSE)" is an equilibrium in which r_t and W_t are constant all t and in which Q, K, and E grow geometrically with factor $g \cdot n$. As stated, this section focuses exclusively on steady–state equilibria.

Calibration. Table 1 presents our mortality and labor supply data. We use two schedules for q_s, one averaging 1995 United States mortality rates for men and women, and a second based on 1920 rates.[10] Column 4 provides recent data on relative earnings at different ages. Columns 5–6 multiply column 4 by participation rates. We set e_s for our "1995" and "1920" simulations from columns 5–6. Notice that while survival rates past age 65 were much lower in 1920, participation rates among survivors were much higher. Notice also that the relative e_s for different ages within columns 5–6 make a difference in this paper, but the absolute levels do not.[11]

Table 2 presents base–case values for other parameters. Labor's share of output equals $1-\alpha$. Letting 1995 wages and salaries from *The Economic Report of the President* (1999) be A, proprietor's incomes be B, and all income be C, we derive α from

$$1-\alpha = \frac{A+(1-\alpha)\cdot B}{C}.$$

The real interest rate implied by Huggett's (1996) capital share, empirical capital–to–output ratio, and depreciation rate is .06. According to (33),

$$r_{1995}+\mu = \frac{\alpha \cdot Q_{1995}}{K_{1995}}.$$

Using the 1995 GDP and stock of business inventories from *The Economic Report of the President* (1999), and combining the inventory stock with the 1995 fixed private capital stock from *The Survey of Current Business* (1997, p.38), the formula above yields $r_{1995} = 1.059$ if we set $\mu = .08$. The latter is our choice in Table 2. K_t^D/Q_t for 1995 comes from the same two sources; for 1920, the GDP comes from *Historical Statistics of the United States* (1975) (and the stock of durables is extrapolated from 1925).

We employ one value of n reflecting U.S. Census data for 1970–90, and a second based on 1900–1920. The rate of technological progress has fluctuated during the century, and our base–case models simply set our progress factor, g, to 1.01.

In Huggett (1996), each household's x changes from year to year. Using his initial and annual shocks and autocorrelation and our 1995 e_s, n, and g, one can simulate the U.S. earnings distribution. Using our e_s, n, g, and employing this paper's assumption that each adult's x is constant throughout his life, for each σ_x^2 (recall (25)) we can generate a second simulated distribution. Table 2's σ_x^2 equates the Gini coefficient for the second simulated distribution to Huggett's.

[10] According to these figures, life expectancy at age 20 in 1995 was 75.8, whereas in 1920 it was 64.1.

[11] One could speculate that lifetime earnings profiles were flatter in 1920 than 1995 because of lower education attainment. That would have tended to narrow differences in life–cycle saving between the two periods — but it is not taken into account in this paper.

Table 2's aggregative social security benefits come from *Social Security Bulletin* (1997, p.61). The corresponding 1995 tax rate (see (34)) is $\tau_{ss} = .061$.[12] The U.S. had no social security system in 1920. B_t and G_t for 1995 come from *The Economic Report of the President* (1999), referring to Federal debt and spending, respectively; for 1920 they come (with extrapolation when necessary) from *Historical Statistics of the United States* (1975). We derive tax rate τ from (35); in the 1995 base case it is .237, and for 1920 it is .108.

Table 2's γ comes from Huggett (1996). (As this parameter serves mainly to scale β in our setup, we do not present simulations below for alternative values of it.) Recalling utility functions $u(c_s, s)$ and $u^k(c_{ks}, s)$, first–order conditions show that a retiree will have υ times as much consumption as a nonretiree, *cet. par.*, and a minor child will have ω times as much consumption as his parent. Retirees tend to have lower consumption needs (not having work–related expense for clothing and transportation, being able to consume services at off–peak hours, etc.). For example, a recent TIAA–CREF brochure suggests "you'll need 60 to 90 percent of your current income in retirement, adjusted for inflation, to maintain the lifestyle you now lead"; a recent *Reader's Digest* article on retirement planning states, "Many financial planners say it will take 70 to 80 percent of your current income to maintain your standard of living when you retire"; and, Mariger (1986) econometrically estimates that individual consumption falls 50% at retirement. Our base case sets $\upsilon = .75$. Tobin (1967) suggests values for ω ranging from .2 to .7; Mariger's (1986) point estimate is $\omega = .3$; and, empirically derived scales for consumption needs of 4–person households relative to 2–person in Burkhauser *et al.* (1996) suggest ratios of 1.34–1.42. Our base case follows Mariger.

Turning to β, the first–order condition for adult consumption at consecutive ages yields

$$(\beta \cdot R)^{1/(1-\gamma)} \cdot c_s \leq c_{s+1}, \tag{38}$$

with equality when the liquidity constraint $a' \geq 0$ does not bind. Tables from the 1984–97 *U.S. Consumer Expenditure Survey* (see http://stats.bls.gov/csxhome.htm) provide data on consumption at different ages. Since the survey does not impute service flows to owner occupied houses, for each year we scale survey amounts (less mortgages and repairs on owner occupied houses) to NIPA aggregate consumption minus aggregate housing service flows for owner occupied houses, then we allocate the NIPA aggregate service flow from owner occupied houses to survey age brackets in proportion to average housing values within the brackets (as given in the survey).[13] Finally, we extrapolate to individual ages and convert to constant dollars with the NIPA personal consumption deflator. The average ratio of time–$(t+1)$ household consumption at age $s+1$ to time–t consumption at age s for households of age 30–39 is 1.0257 — where we include only these ages out of fear that in practice liquidity constraints bind for earlier ages and minor children begin leaving home

[12] Note that our social security benefits refer only to old–age and survivors insurance, not to disability insurance.

[13] Another potential problem, of course, is that the survey measures purchases of consumer durables rather than service flows from them. See, for example, Modigliani (1988).

at later ages.[14] Recalling (38), our base–case β follows from

$$(\beta \cdot R)^{1/(1-\gamma)} = 1.0257,$$

where $R = 1 + r \cdot (1 - \tau)$ with $r = r_{1995} = .059$, $\tau = \tau_{1995} = .237$, and we disregard (at this point) mortality.

Sections 2–5 and the envelope theorem show that a steady state with positive inheritances requires

$$R^{20} \cdot \xi \cdot g^{(\gamma-1)\cdot 20} = 1. \tag{39}$$

Our base–case simulations compute $\xi = .5588$ from this formula using $R = 1 + .059 \cdot (1 - .237)$ and $g = 1.01$.[15]

For future reference, the last rows of Table 2 display 1995 ratios of aggregative household net worth to total factor payments for labor, and private capital other than consumer durables to GDP, using the data sources explained above.

Simulations. Tables 3–5 present simulation results.

The computational steps for row 1 of Tables 3–4 is as follows. (i) Compute r and W from (33), assuming Table 2's empirical K_t/Q_t. (ii) After dividing (35) through by Q_t, one can deduce τ. (iii) Then solve the life–cycle problem of a nondynastic household with $x = 1$. Because of homothetic preferences, different x values affect net worth at all ages proportionately. Taking a ratio of average cross–sectional nondynastic net worth to average effective labor, compare with the second to the last row of Table 2 to compute the nondynastic share of total private wealth. (iv) For any choice of λ, dynastic wealth supplies the remaining part of total wealth, and we can compute the cross–sectional distribution of wealth, including nondynastic and dynastic households. Huggett (1996) and Wolff (1987) suggest the Gini coefficient for the actual U.S. distribution of wealth in the late 1980s was .72, the share of the top 1% of wealth holders was .28, and the share of the top 5% was .49. We compute the λ such that the sum of squared deviations between the simulated and actual values of the latter three statistics is minimized. Table 3 reports the λ, and Table 4 presents distributional outcomes.

There are two additional details in our simulations of the distribution of wealth. First, our algorithm must specify the age at which dynastic intergenerational transfers actually take place — though this makes no difference to aggregate wealth, it does affect distributional statistics. Laitner (1997) argues that parents may withhold their transfers as long as possible to reduce children's scope for strategic behavior (for instance, spending their transfer and asking for more). Our computations follow that spirit. For each dynastic household, trace the oldest surviving ancestor, "the patriarch." Assume that he holds the

[14] For comparison, Auerbach and Kotlikoff's (1987, fig.5.2) consumption has an average growth factor of about 1.0105, and Huggett's (1996) factor ranges from 1.017 to 1.035, depending on one's choice of γ. With our 1.0257 factor, consumption is 2.4 times as large at age 65 as at age 30.

[15] Using a somewhat different model, Nishiyama (2000, table 8–9) derives estimates .51 and .58 for a parameter analogous to $\xi/\beta^{20} = .63$ in the present paper.

dynasty's wealth. On the birth of each descendant, he creates a "trust" account in that descendant's name, funding it with the present value of the amount the descendant will inherit less what the descendent himself will bequeath, and annuitizing on the recipient's life. Each year the account makes the minimal *inter vivos* transfers to the descendant necessary for him to implement his lifetime consumption program derived from (26)–(28). When the patriarch dies, the balances for all trust accounts pass to the oldest surviving descendants in each line radiating from the patriarch. The model's distribution of wealth reflects this: as long as a parent, for example, holds trust accounts for his children, the balances of the accounts are counted as part of the parent's (rather than the children's) net worth. Analogously, the parent himself does not inherit the principal of his own inheritance until his parents, grandparents, and great grandparents are deceased.

Second, although the model fully determines the SSE distribution of wealth among nondynastic households, the distribution of dynastic wealth — though not its aggregate amount — is indeterminate. This section assumes that the distribution of wealth among dynastic households of each age is proportional to their "abilities." This is strictly analogous to Sections 2–5: the dynastic wealth distribution there was indeterminate, but we assumed that all — having the same ability — had the same wealth.

For row 1 of Table 3, nondynastic households explain about 68% of the economy's total wealth. As in Huggett (1996), the distribution of nondynastic wealth matches the empirical Gini quite well but understates the concentration at the top of the distribution by a wide margin — see Table 4. When we add dynastic households, the match with the upper tail is very close, although the Gini overshoots its corresponding actual value. The fraction of dynastic households yielding the best fit is quite low, $\lambda = .083$. Table 3 shows that the dynastic households have on average almost 7 times as much net worth as purely life–cycle households, despite the fact that earnings distributions are the same for both groups. Table 5 compares average net worth at different ages. The disparity in net worth becomes very great for the elderly: purely life–cycle households begin decumulating net worth around retirement age, but dynastic households build wealth, through inheritances and interest accumulations, until death. The average inheritance of a dynastic household, in present value at age 20, is about \$450,000.

Rows 2–5 follow the same steps, using parameters which deviate from the 1995 base case as shown in Table 3. When we increase the importance of retirement consumption or decrease the early in life burden of raising children, the share of nondynastic private wealth rises about 10 percent. The requisite λ then drops precipitously — as dynastic wealth, with a smaller overall role, must be more concentrated to match the actual wealth distribution's upper tail.

Rows 6–10 of Tables 3–4 present comparative–static results. Instead of calibrating λ, β, ξ, and μ to match empirical distributional statistics, consumption growth with age, the interest rate, and wealth–to–earnings ratio, we fix these four parameters from row 1's simulation. In row 6, we change K_t^D/Q_t, n, SSB_t/Q_t, B_t/Q_t, G_t/Q_t, and our mortality and labor–force participation schedules (see Table 1) to 1920 levels. Then we compute the new steady–state interest rate, consumption–growth–with–age rate, wealth–to–earnings ratio, τ, τ_{ss}, and distributional statistics. Row 7 repeats the computations with only mortality and participation changing to 1920 levels (i.e., with K_t^D/Q_t, n, SSB_t/Q_t, B_t/Q_t,

and G_t/Q_t remaining at 1995 values). Row 8 returns to 1995 levels of K_t^D/Q_t, n, SSB_t/Q_t, B_t/Q_t, G_t/Q_t, and mortality and participation, but it raises SSB_t by 50%. Then it computes a new steady–state interest rate, consumption–growth–with–age rate, wealth–to–earnings ratio, τ, τ_{ss}, and wealth distribution. Rows 9–10 do the same after raising B_t by 50% (for row 9) and the rate of technological progress by 100% (for row 10).

Row 6 of Tables 3–4 bears out the theoretical analysis of Section 3. With shorter lives and higher labor force participation among survivors, the life–cycle saving of nondynastic households in 1920 accounts for less than one–half of private wealth, as opposed to two–thirds in 1995, and average dynastic wealth is 13 times as high as nondynastic wealth, as opposed to 7 times in 1995. In the simulation for 1920, the Gini coefficient for the overall distribution of wealth is .87 and the share of the top 1% is .35, while for 1995 the simulation's Gini is .79 and the share of the top 1% is .27. The net of tax interest rate, nevertheless, is the same for both years. Row 7 shows the comparison would be even starker had not social security, national debt, and government spending changed between 1920 and 1995: the share of nondynastic wealth in 1920 would have been only .29, average dynastic net worth would have been 33 times as high as nondynastic wealth, the Gini of the wealth distribution would have been .91, and the share of the top 1% would have been .45.

Rows 8–9 of Tables 3–4 confirm Section 4's theoretical analysis of how social security and national debt can raise the relative importance of dynastic saving and thereby increase wealth inequality. In row 8, a 50% increase in the size of the (1995) social security system lowers the share of private wealth held by nondynastic households from .68 to .55, raising the Gini for the steady–state wealth distribution from .79 to .83, and raising the share of wealth held by the top 1% from .27 to .35. Evidently the effects of social security changes of this magnitude would be almost as great as the combined demographic and governmental changes between 1920 and 1995. Row 9 shows a 50% increase in the steady–state national debt–to–output ratio is less potent: such a change reduces the share of net worth of nondynastic households from .68 to .63, raises the wealth Gini from .79 to .80, and raises the share of the top 1% of wealth holders from .27 to .30.

Row 10 of Tables 3–4 is consistent with Proposition 6 of Section 5. Row 10 shows that an increase in the rate of technological progress raises the steady–state interest rate substantially and reduces wealth inequality. The absolute change in inequality in this last case, however, is not as great as demographic and social security outcomes in rows 6–8: the Gini for the simulated distribution of wealth falls from .79 with $g = 1.01$ to .78 with $g = 1.02$, and the share of the top 1% falls from .27 to .24.

7. Conclusion

This paper presents a life–cycle model augmented with intentional bequests. A principal feature of the model is heterogeneity of preferences: most households feel no obligations to their grown children; however, a small fraction of the population has dynastic, or "altruistic," sentiments.

Despite its simplicity and tractability, the model has a number of interesting implications. (i) It shows that lengthening life spans and falling rates of labor force participation among older age groups could have increased life–cycle saving in the U.S. and other countries over the course of the twentieth century without affecting long term interest rates or

aggregative capital to labor ratios. (ii) It shows how the same demographic phenomena might have played a role in decreasing the degree of inequality in cross sectional distributions of wealth over the same time period. (iii) It shows that government policies regarding national debt and social security can influence wealth inequality, as can changes in the rate of technological progress. (iv) It shows why so–called Ricardian neutrality might hold in the long run yet not in the shorter term.

Recent empirical work has tended to yield what seems to be at most quite mild support for the altruistic model of household behavior. The present paper offers possible interpretations, however. For instance, Altonji *et al.* (1992) show that intergenerational linkages apparently are far from universal, and evidence in Laitner and Juster (1996) seems to imply that linkages which do exist are hard to predict on the basis of economic variables. Both findings are consistent with this paper's analysis. Even more recently, regressions in Altonji *et al.* (1997) and Laitner and Ohlsson (2001) based on *inter vivos* gifts and inheritances, respectively, in the *Panel Study of Income Dynamics* derive coefficient estimates which conform in sign to the altruistic model but are much smaller in magnitude than the theory predicts. In the present paper, the calibrated examples fit distributional data best when dynastic behavior is rare, perhaps manifested by 10% of all households or less. On the one hand, this suggests that intergenerational transfers evident in general data sets, where their frequency is 20–40%, may represent a mixture of altruistic and nonaltruistic behavior — and that the mixing may bias econometric results. On the other hand, our simulations imply substantial differences between the net worth of dynastic and nondynastic households, perhaps, in turn, indicating a need for special data sources able to capture the behavior of very wealthy households.

Appendix

This appendix describes the simulations for Fig. 6 in Section 4.

First, we develop a system of three equations determining the evolution of a state vector $\mathbf{z}_t \equiv (\tilde{K}_t, \tilde{s}^d_{1,t-1}, \tilde{i}_t)$, where $\tilde{K}_t \equiv K_t/g^t$, $\tilde{s}^d_{1,t-1} \equiv s^d_{1,t-1}/g^{t-1}$, $\tilde{i}_t \equiv i_t/g^t$. Without loss of generality, set $N = 1$. One equation comes from an accounting relation for dynastic families,

$$R_t \cdot s^d_{1,t-1} = c^d_{2,t-1} + i_t; \tag{A1}$$

the marginal condition for such a family in its second period of life,

$$\frac{\theta}{c^d_{2,t-1}} = \xi \cdot \frac{1-\theta}{c^d_{1t}} \Longleftrightarrow c^d_{1t} = \frac{1-\theta}{\theta} \cdot \xi \cdot c^d_{2,t-1}; \tag{A2}$$

and first period of life accounting,

$$c^d_{1t} = W_t \cdot g^t - \tau_t + i_t - s^d_{1t}. \tag{A3}$$

Combining the three, we have

$$W_t \cdot g^t - \tau_t + i_t - s^d_{1t} = \frac{1-\theta}{\theta} \cdot \xi \cdot [R_t \cdot s^d_{1,t-1} - i_t]. \tag{A4}$$

Using $B_t = B_0 \cdot g^t$ and government budget constraint (23) to eliminate τ_t, using (11) to characterize R_t and W_t from K_t, and then dividing through by g^t, this yields an equation in $\mathbf{z}_t$ and $\mathbf{z}_{t+1}$.

The second equation comes directly from Section 4:

$$s^d_{1t} = \theta \cdot [W_t \cdot g^t + i_t - \tau_t] + \frac{(1-\theta) \cdot i_{t+1}}{R_{t+1}}. \tag{A5}$$

The final equation comes from (24):

$$K_{t+1} + B_{t+1} = (1-\lambda) \cdot \theta \cdot [W_t \cdot g^t - \tau_t] + \lambda \cdot s^d_{1t}. \tag{A6}$$

For Fig. 6, differentiate the detrended system generated from (A4)–(A6) with respect to B_0, producing a difference equation

$$\frac{d\mathbf{z}_{t+1}}{dB_0} = \mathbf{M} \cdot \frac{d\mathbf{z}_t}{dB_0} + \mathbf{m}, \tag{A7}$$

where $\mathbf{M}$ is a 3×3 matrix and $\mathbf{m}$ a 3–element vector. Evaluate the terms in $\mathbf{M}$ and $\mathbf{m}$ at the old steady state, producing a linearization there. As we start, history provides initial values for $\tilde{K}_t$ and $\tilde{s}^d_{1.t-1}$, but not for $\tilde{i}_t$; thus, under rational expectations, a linearized version of the dynamic system needs exactly two eigenvalues of absolute value less than one to avoid indeterminacy and instability — i.e., to achieve so–called "saddlepoint stability." That turns out to be the case for the example, the eigenvalues being 1.39, 0.65, and 0.35.

Simulation of (A7) yields, for example, dK_t/dB_0 all $t = 0, 1, 2, ...$ Letting K_t be the capital in the old steady state, Fig. 6 graphs

$$\frac{dK_t}{dB_0} \cdot \frac{B_0}{K_t} \cdot [\frac{B_{00} - B_0}{B_0}],$$

with $(B_{00} - B_0)/B_0$ set to 1%, and B_{00} the new time–0 government debt. See Laitner (1990), for instance, for more discussion of this general methodology.

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Table 1. Household Data: Survival Fractions, Labor Force Participation, and Experiential Human Capital

Age s	1995 Fraction 20 Yearolds Remaining Alive	1920 Fraction 20 Yearolds Remaining Alive	Life–Cycle Earnings	Life–Cycle Earnings × Participation Rates 1995	Life–Cycle Earnings × Participation Rates 1920
20	1.00	1.00	5,814	4,678	5,174
25	.99	.97	13,341	11,615	12,176
30	.99	.94	19,333	17,973	18,085
35	.98	.91	23,323	21,601	22,269
40	.97	.88	26,666	24,566	25,133
45	.96	.84	28,967	26,179	26,947
50	.94	.79	28,885	25,320	26,517
55	.91	.74	26,941	20,545	24,435
60	.87	.67	23,763	14,984	21,553
65	.82	.57	12,719	4,274	11,536
70	.73	.46	6,396	1,477	3,988
75	.63	.33	4,788	603	1,629
80	.49	.20	3,264	69	185
85	.32	.09	1,740	0	0
90	.13	.03	215	0	0
95	.02	.00	0	0	0
100	.00	.00	0	0	0

Source: Column 2: average death rates 1990, *Statistical Abstract of the United States* (1998, p.95).
Column 3: death rates 1920, *Historical Statistics of the United States* (1975, p.60).
Column 4: Median earnings men 1990,
Social Security Bulletin Annual Statistical Supplement (1993, p.165).
Column 5: column 4 multiplied by 1995 male labor force participation rates,
Statistical Abstract of the United States (1997, p.397).
Column 6: column 4 multiplied by 1920 male labor force participation rates,
Historical Statistics of the United States (1975, p.132).

Table 2. Base–Case Parameters and Empirical Ratios

Base–Case Parameters	Year	
	1995	1920
α	.3251	.3251
μ	.0800	.0800
K_t^D/Q_t	.3196	.3437
n	1.0101	1.0166
g	1.0100	1.0100
σ_x^2	.5344	.5344
SSB_t/Q_t	.0410	.0000
B_t/Q_t	.4532	.2634
G_t/Q_t	.1866	.0824
γ	-.5000	-.5000
υ	.7500	.7500
ω	.3000	.3000
β	.9941	.9941
ξ	.5588	.5588
1995 Empirical Ratios		
$[K_t + K_t^D + B_t]/[(1-\alpha)\cdot Q_t]$	4.6102	
K_t/Q_t	2.3386	

Source: see text.

Table 3. Simulated Fraction of Dynastic Family Lines

Trial	Simu–lation Year	Deviation from Year's Base–Case Para–meters[a]	Non–dynastic Share of Total Private Wealth	Ratio Average Dynastic to Non–dynastic Wealth	R Net of Tax	R Gross of Tax	Frac–tion Dynas–tic Fami–lies: λ
Base–Case 1995 and Sensitivity Analysis							
1	1995	none	.678	6.702	1.045	1.059	.083
2	1995	$\upsilon = .85$	.761	12.791	1.045	1.059	.027
3	1995	$\omega = .20$	.740	9.786	1.045	1.059	.040
4	1995	$\sigma_x^2 = .6344$	.678	5.734	1.045	1.059	.100
5	1995	$\mu = .06$	.619	5.742	1.061	1.079	.130
Comparative Statics: β, ξ, and λ from Trial 1							
6	1920	none	.494	13.341	1.045	1.051	.083
7	1920	trial 1 parameters; 1920 mortality and participation rates	.285	33.268	1.045	1.059	.083
8	1995	$SSB_t/Q_t = .062$	.551	10.822	1.045	1.059	.083
9	1995	$B_t/Q_t = .680$	.627	8.182	1.045	1.060	.083
10	1995	$g = 1.02$	.722	5.631	1.061	1.079	.083

a. See Tables 1–2.

Table 4. Simulated Distribution of Wealth

Trial[a]	Nondynastic Households				All Households			
	Gini	Share Top 1%	Share Top 5%	Fraction zero wealth	Gini	Share Top 1%	Share Top 5%	Fraction zero wealth
Base–Case 1995 and Sensitivity Analysis								
1	.690	.098	.300	.284	.787	.267	.505	.303
2	.687	.098	.299	.284	.762	.285	.462	.295
3	.672	.095	.292	.263	.758	.283	.469	.281
4	.705	.109	.322	.284	.793	.257	.509	.304
5	.703	.101	.307	.363	.811	.256	.537	.408
Comparative Statics: β, ξ, and λ from Trial 1								
6	.778	.127	.376	.550	.871	.353	.650	.547
7	.743	.111	.337	.508	.907	.448	.785	.516
8	.693	.099	.301	.324	.827	.349	.599	.356
9	.690	.098	.300	.284	.803	.301	.544	.319
10	.695	.099	.303	.324	.777	.239	.474	.339

a. Parameter values as in Table 3.

Table 5. Average Household Net Worth (dollars) at Selected Ages: 1995 Base–Case Parameters

Age	Nondynastic Household Net Worth	Dynastic Household Net Worth	Overall Household Net Worth
20	0	1,106	92
25	0	3,477	290
30	0	10,471	873
35	1,111	28,266	3,375
40	10,267	68,052	15,084
45	66,852	161,454	74,739
50	138,403	344,627	155,595
55	204,771	623,045	239,642
60	246,880	1,010,905	310,576
65	231,846	1,464,889	334,643
70	221,226	2,122,170	379,705
75	209,734	2,772,003	423,347
80	181,234	3,181,128	431,331
85	136,147	3,331,539	402,543
90	88,396	3,637,942	384,317
95	44,001	4,090,848	381,381

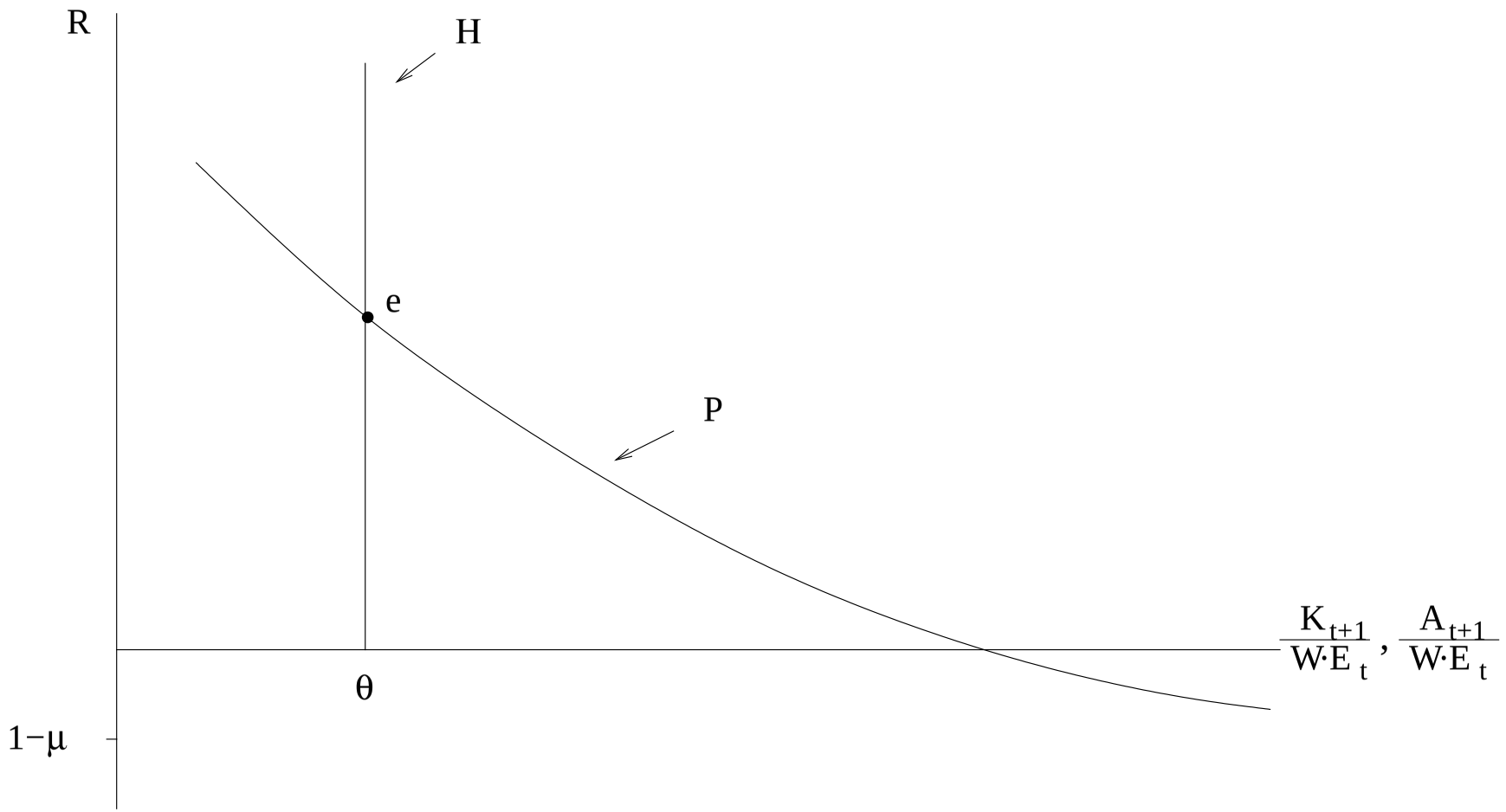


Fig. 1: The SSE demand (P) and supply (H) of wealth in the pure life−cycle case

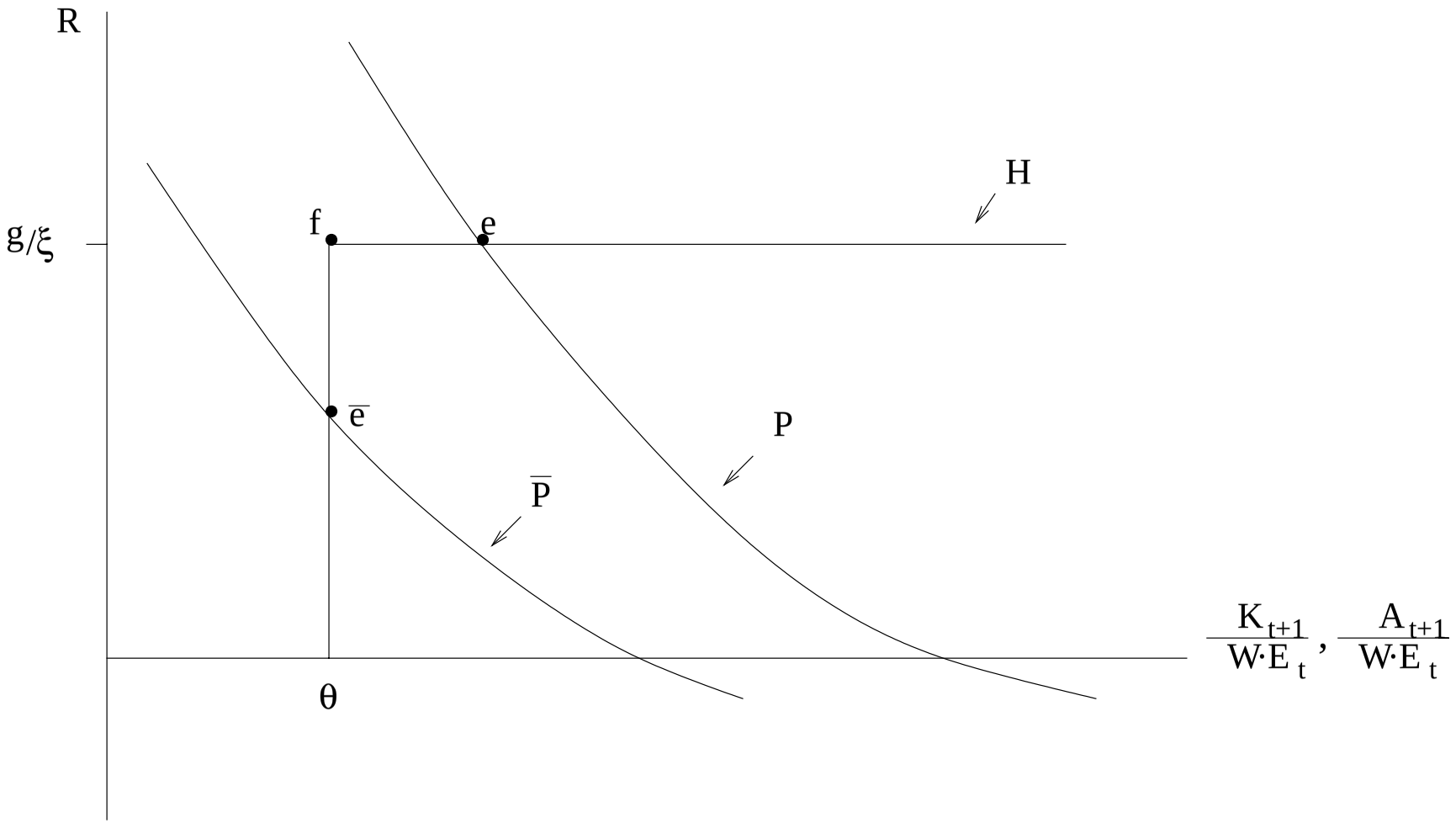


Fig. 2: The SSE demand and supply of wealth with dynastic saving

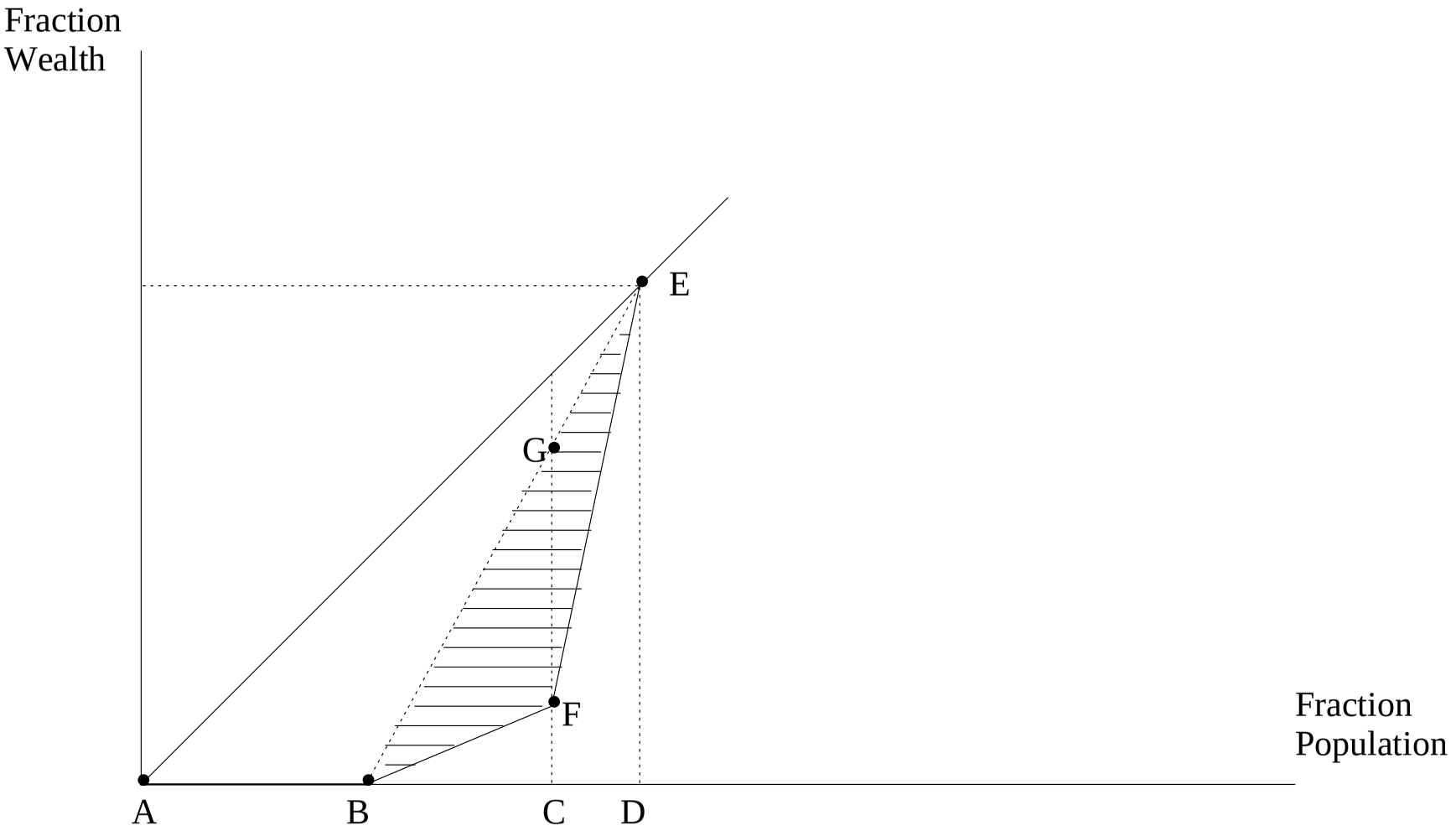


Fig. 3: The Lorenz curve (ABFE) for the cross−sectional distribution of wealth

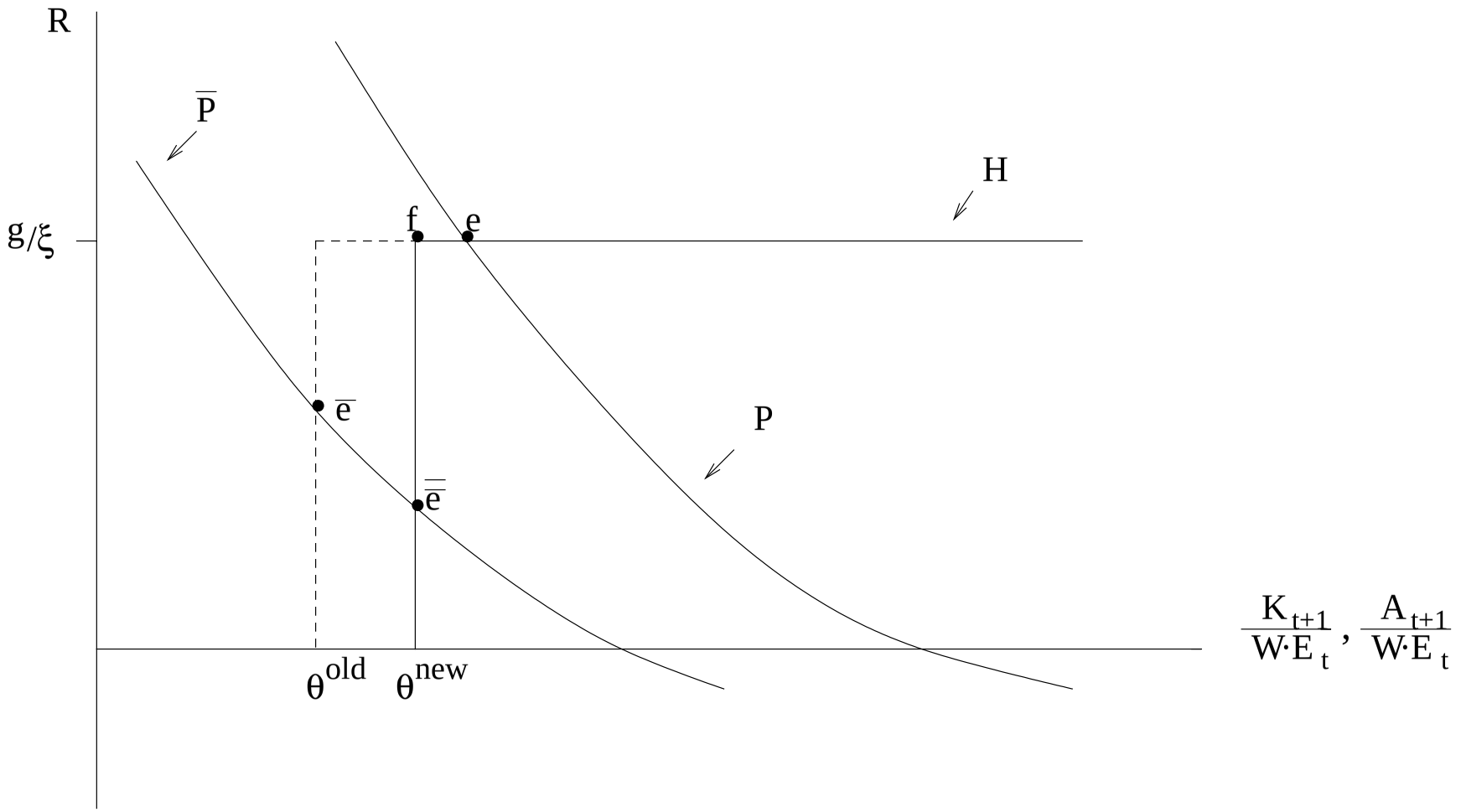


Fig. 4: The H−curve shifts because of an increase in longevity

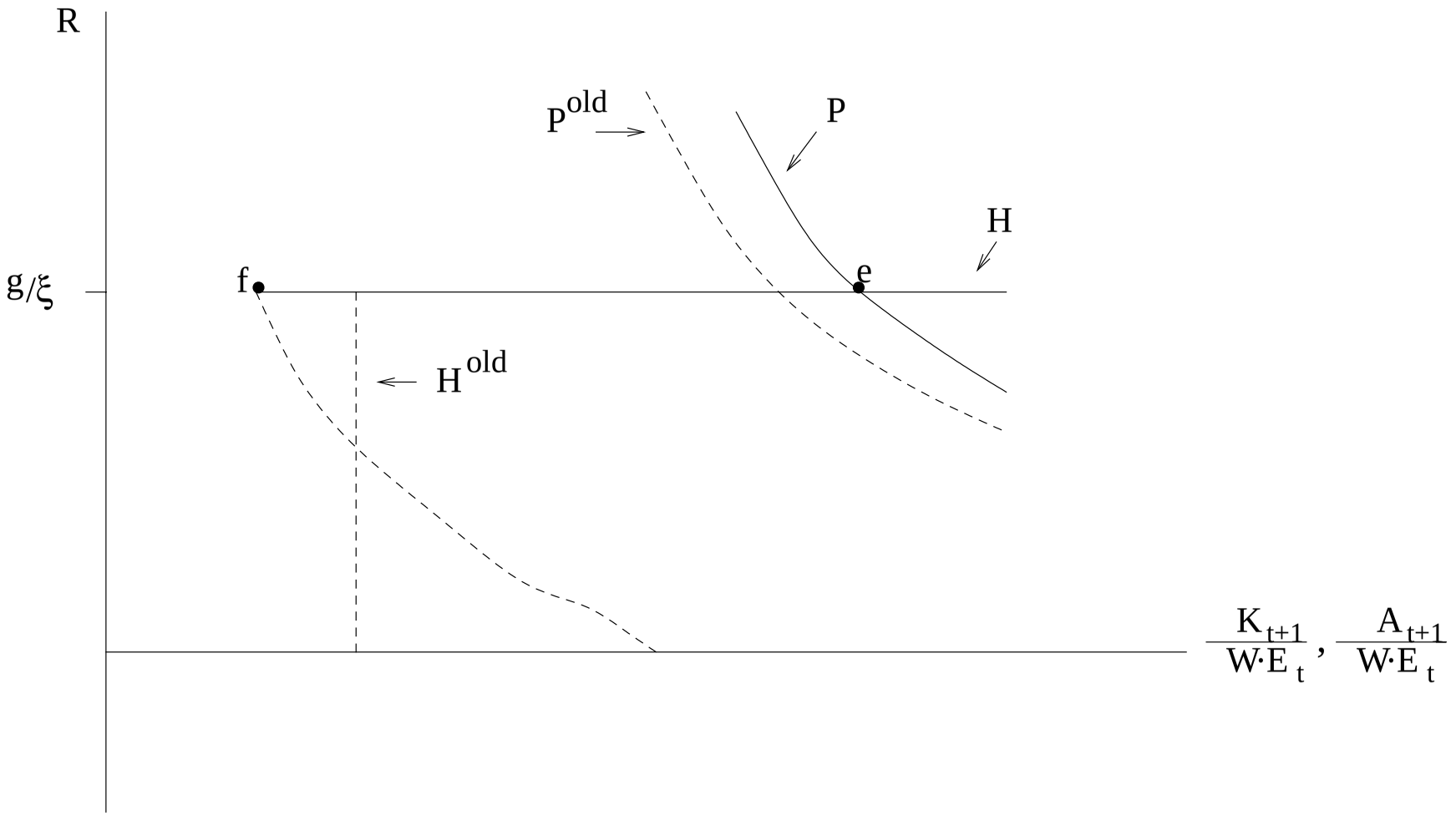
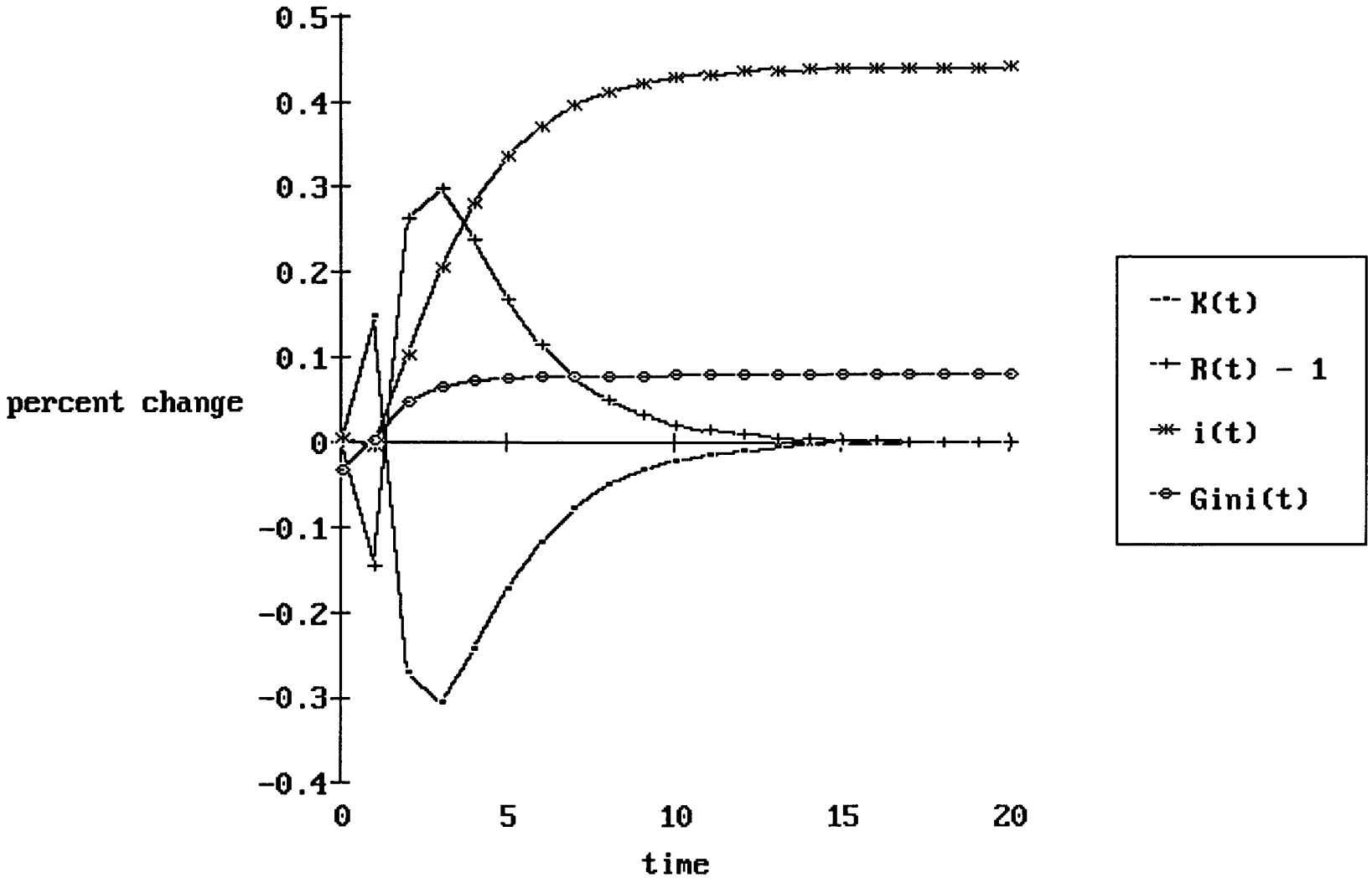


Fig. 5: The effect of a national debt on the economy's SSE

Fig. 6: Changes following a 1% increase in B(1)



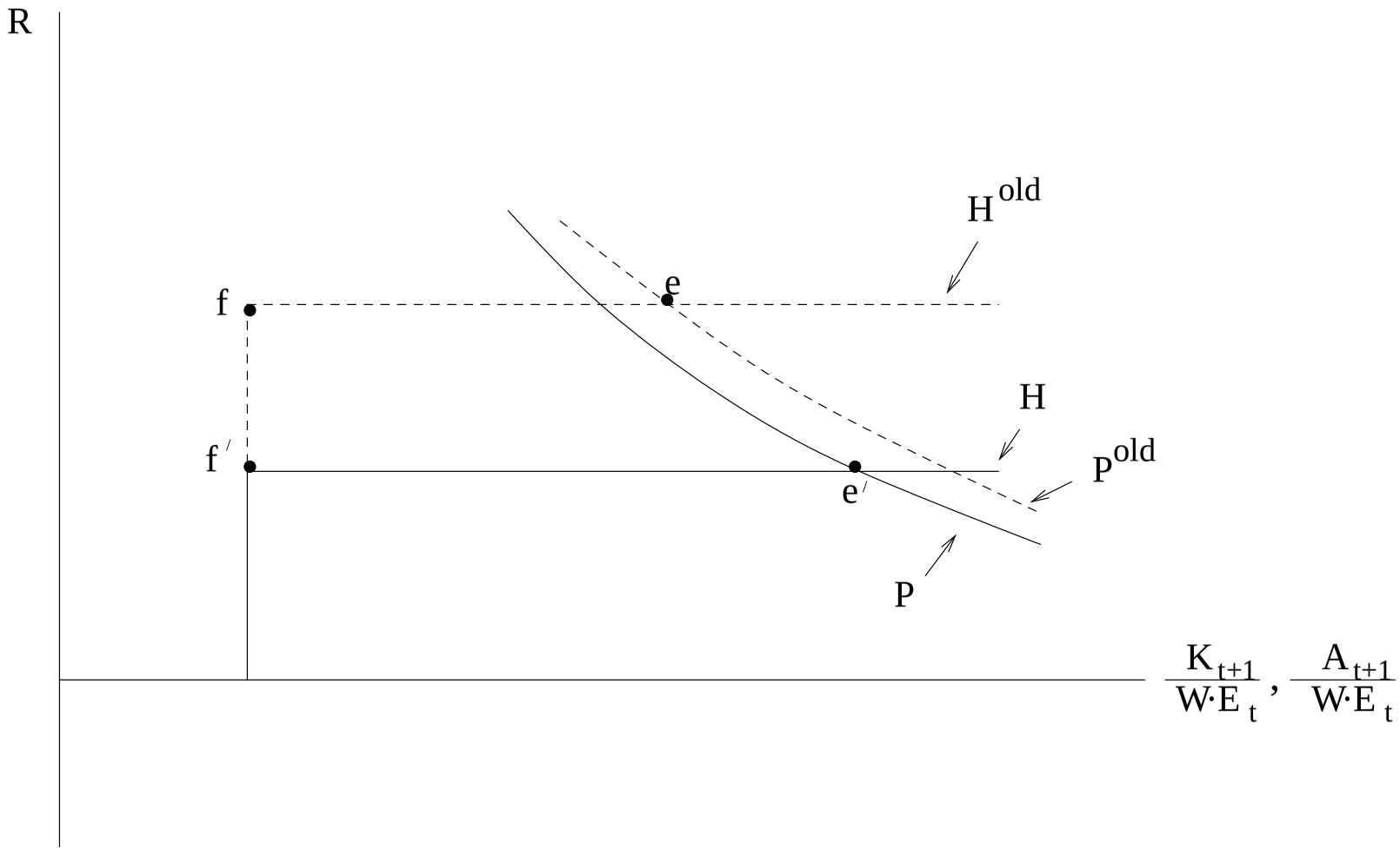


Fig. 7: The effect of slower technological change on the economy's SSE