

## MORTALITY CONTINGENT CLAIMS: IMPACT OF CAPITAL MARKET, INCOME, AND INTEREST RATE RISK

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In this paper, we analyze how income, stock market, and interest rate risk affect the purchase of life insurance and pension annuities over the life cycle in a continuous time setting. We expand prior literature on pension annuities and life insurance by simultaneously considering income, stock market, and interest rate risk as the main sources of uncertainty over the life cycle apart from mortality risk. Mortality risk has two different outcomes. On the one hand, the investor can run out of savings and fall into poverty before dying. In this case, the investor can neither consume nor bequeath her heirs (longevity risk). On the other hand, the investor can die too early without consuming enough of her savings if she has no bequest motive, or she can perish without having built up sufficient wealth to leave an appropriate legacy for her heirs. The latter is also known as brevity risk.

While in the past only wealthy individuals were concerned with personal financial planning, today everyone has to accumulate wealth and manage their assets effectively to meet the consumption and bequest liabilities that arise during their lifetime. Personal finance becomes critical for everyone's individual welfare in light of the demographic shifts eroding and undermining public PAYGO systems. Until recently, several employers have taken on longevity and investment risk by providing defined-benefit solutions to their workers. Today's employees have to increasingly bear the investment risk when electing a defined-contribution plan from their employer's menu. Ideal candidates for the efficient management of consumption- and bequest-liabilities are pension annuities and life insurance policies with guaranteed payouts.

Both products have appealing payoff characteristics since their pricing is either contingent on surviving (pension annuities) or on dying (life insurance). Compared to an investment in mutual funds, the consumption possibilities are higher when annuitizing, while the bequest potential increases when purchasing life insurance. If the individual annuitizes, he surrenders bequest potential, while if he purchases life insurance, he gives up consumption possibilities. If the investor purchases life insurance, the premium is taken out of the investor's financial wealth. The investor controls his legacy by finding the right balance between savings and life insurance early in life and seeks the appropriate split between financial wealth and pension annuities during the later stage of his life cycle.

## **RESULTS:**

We solve a life-cycle model with life insurance and pension annuities analytically for the complete market case. Our model assumes a stochastic wage earner with constant relative risk aversion (CRRA) preferences whose lifetime is random. The investor has to decide among short and long positions in life insurance, stocks, bonds,

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and money market investment when facing a risky stock market and interest rate risk. We find an analytical solution for the complete market case in which human capital is exactly priced. We also derive some numerical insight into a realistically calibrated case when income is unspanned. An illustrative analysis shows when the wage earner's demand for life insurance switches to the demand for annuities. The results can be summarized as follows:

Overall Demand for Life Contingent Products with Guaranteed Payouts: Life insurance policies play an important role for investors with relatively small levels of savings. The higher the level of savings, the more is the investor induced to switch to short positions because he has sufficient financial wealth levels in order to bequeath his heirs. The overall insurance demand changes considerably over the life cycle. The total fraction used for life insurance purchases and pension annuities early in life is still limited. This is because life insurance premiums are relatively cheap. Yet, pension annuities are relatively expensive because they limit the bequest potential, which is not sufficiently backed by financial savings. When the investor becomes older, he purchases substantially more pension annuities.

Labor Income Risk: Even though the selection of life insurance and pension annuities is a challenging task in a world where mortality is the sole source of risk, there are many other sources of risk influencing both the timing and the extent to which life insurance and pension annuities are purchased. Labor income is probably the most influential exogenous factor determining the timing of pension annuities. By the same token, labor income also influences the level of savings and therefore the overall demand for life insurance. Not only does labor income have a considerable impact on the purchase of life insurance and pension annuities, but the influence on the asset allocation within the investor's financial wealth is also considerable. Several of these studies find labor income to be more closely related to bonds than to stocks. Therefore, human capital acts partly as a substitute for bonds. In turn, the optimal stock fraction of financial wealth decreases over the life cycle as the fraction of human capital declines in the investor's total augmented wealth. Augmented wealth is composed of both financial wealth and human capital.

Interest Rate and Investment Risk: Interest rate dynamics are important for determining the value of the investor's human capital. We find it particularly interesting to analyze its impact on the decision of when to buy pension annuities and when to purchase life insurance. Correlation between the innovations of the stock return and the income growth had a limited impact on the overall asset allocation. Consistent with previous research, we show that correlation particularly matters if the investor faces a stochastic investment opportunity set with correlated interest rate risk.

Timing of Life Insurance and Pension Annuities: The optimal life annuity demand depends on all state variables (wealth, income, short rate, and age) as well as all parameters under consideration. The insurance demand is particularly dependent on age. The older the individual, the higher the hazard rate, and the greater is the absolute demand for life insurance products. The lower the investor's human capital and the higher her financial wealth, the more likely the investor shifts from life insurance into pension annuities. We find a substantial impact of financial wealth with respect to the insurance rules, but we discover a considerably small influence of the short rate on the demand of life insurance if we reasonably calibrate our asset and income model.

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